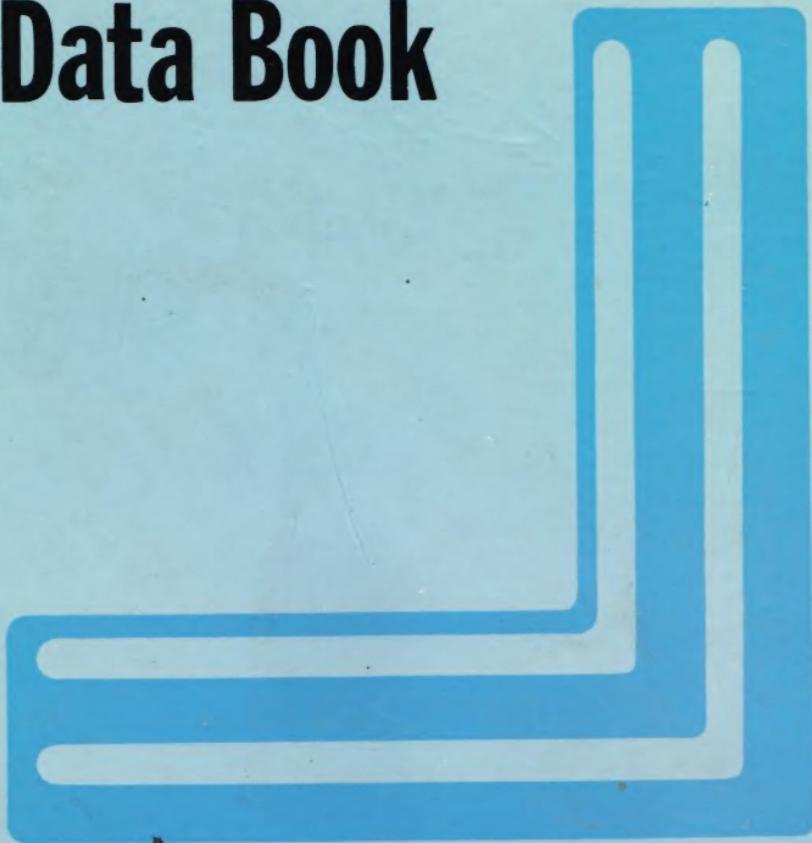


An Engineering Data Book



Edited by
A J Munday and R A Farrar

REASON FOR PUBLICATION

This book has been produced to provide a pocketable source of data for students pursuing most Engineering Degree Courses, and for use in examinations. * It was not designed for use in Electrical Degree Courses.

It differs from other data books in two respects; it has a comprehensive key-word index and a symbols index in order that users may find data efficiently.

A Professional Engineer should not rely on the memory of facts for use in a design situation, until their frequent use has committed them permanently and accurately to the memory. Until that happy time is reached a data book makes life easier, and makes the permanent retention of accurate facts more likely.

The editors hope that no errors exist but cannot guarantee the accuracy of the data. If you find any errors the editors would appreciate your comments for inclusion in further editions.

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- v) Other engineering data sources too numerous to mention individually for commonly used values and equations.

* Where permitted by the examining body.

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M

1. UNITS AND ABBREVIATIONS

1.1 Decimal prefixes

symbol	prefix	factor by which unit is multiplied
T	tera	10^{12}
G	giga	10^9
M	mega	10^6
k	kilo	10^3
h	hecto	10^2
da	deca	10
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}

1.2 SI units

(i) Basic units

unit symbol	unit	quantity
m	metre	length
kg	kilogramme	mass
s	second	time
A	ampere	electric current
K	kelvin	thermodynamic temperature
cd	candela	luminous intensity

(ii) Supplementary and derived units

quantity	unit	symbol	equivalent
plane angle	radian	rad	-
force	newton	N	$kg \cdot m/s^2$
work, energy heat	joule	J	N m
power	watt	W	J/s
frequency	hertz	Hz	s ⁻¹
viscosity:			
kinematic		m ² /s	10^6 cSt (centi-stoke)
dynamic		Ns/m ² =Pa s	10^3 cP (centi-poise)
pressure		Pa=N/m ²	Called pascal, Pa
stress		Pa or N/m ²	
<u>electrical units</u>			
potential	volt	V	W/A
resistance	ohm	Ω	V/A
charge	coulomb	C	A s
capacitance	farad	F	A s/V
electric field strength	-	V/m	
electric flux density	-	C/m ²	
<u>magnetic units</u>			
magnetic flux	weber	Wb	$V \cdot s = Nm/A$
inductance	henry	H	$V \cdot s/A = Nm/A^2$
magnetic field strength	-	A/m	
magnetic flux density	tesla	T	$Wb/m^2 = N/(Am)$

1.3 Conversion factors for other units into SI units

Length, area, volume

1 in	= 25.4 mm exactly	$1 \text{ \AA} = 10^{-10} \text{ m}$
1 ft	= 0.3048 m	1 thou=1 mil = 0.001 in = 25.4 μm
1 yd	= 0.914 m	1 micron = 1 μm
1 mile	= 5280 ft = 1.609 km	
1 acre	= 0.4047 ha (Hectare) = 4047 m^2	
1 in ³	= 16.39 cm^3	
1 ft ³	= 0.02832 m^3	
1 gal	= 0.1605 ft ³ = 4546 cm^3 = 4.546 l (Litre)	
1 USgal	= 0.1337 ft ³ = 3785 cm^3	

Velocity

1 mile/h	= 1.467 ft/s = 1.609 km/h = 0.447 m/s
1 knot	= 1.689 ft/s = 1.853 km/h = 0.514 m/s

Mass

1 lb	= 0.4536 kg
1 slug	= 32.17 lb = 14.59 kg
1 ton	= 2240 lb = 1016 kg
1 tonne	= 1 Mg = 1 metric ton

Flowrate

1 ft ³ /s (1 cusec)	= 0.02832 m^3/s
1 gal/min	= 7.577 $10^{-5} \text{ m}^3/\text{s}$ = 0.07577 dm^3/s

Density

1 lb/in ³	= 27.68 g/cm ³
1 lb/ft ³	= 16.02 kg/m ³
1 slug/ft ³	= 515.4 kg/m ³

Thermal conductivity

1 Btu/ft h deg R	= 1.731 J/m s ${}^\circ\text{C}$ = 1.731 W/(mK)
1 cal/cm s deg K	= 418.7 J/m s ${}^\circ\text{C}$ = 418.7 W/(mK)

Force

1 pdl	= 0.1383 N
1 lbf	= 32.17 pdl = 4.448 N
1 tonf	= 9964 N
1 kgf	= 2.205 lbf = 9.807 N
1 dyne	= 10^{-5} N

Torque

1 lbf ft	= 1.356 Nm
1 tonf ft	= 3037 Nm

Power

1 hp	= 550 ft lbf/s = 0.7457 kW
1 ft lbf/s	= 1.356 W

1 metric horsepower (ch, PS) = 0.7355 kW

Energy, work, heat

1 ft lbf	= 1.356 J
1 kW h	= 3.6 MJ
1 Btu	= 778.2 ft lbf = 252 cal = 1055 J
1 cal	= 4.187 J
1 hp h	= 2.685 MJ

Pressure, stress

1 lbf/in ²	= 0.07031 kgf/cm ² = 6895 N/m ²
1 tonf/in ²	= 157.5 kgf/cm ² = 15.44 MN/m ²
1 kgf/cm ²	= 0.09807 MN/m ² = 0.9807 bar
1 kgf/mm ²	= 9.807 MN/m ² = 0.9807 hbar
1 lbf/ft ²	= 47.88 N/m ²
1 ft H ₂ O	= 62.43 lbf/ft ² = 2989 N/m ²
1 in Hg	= 70.73 lbf/ft ² = 3386 N/m ²
1 mm Hg	= 1 torr = 133.3 N/m ²
1 bar	= 14.50 lbf/in ² = 10^5 N/m ²
1 Int atm	= 14.70 lbf/in ² = 10.34 m water = 1.013×10^5 N/m ²
	= 1.013 bar = 760 mm Hg = 101.3 kPa

Dynamic viscosity

1 poise (g/cm s)	=	0.1 kg/m s = 0.1 N s/m ² = 0.1 Pa s
1 kgf s/m ²	=	0.9807 N s/m ²
1 lb/ft h	=	0.4132 mN s/m ²
1 slug/ft s	=	1 lbf s/ft ² = 47.88 N s/m ²
1 lbf s/in ²	=	6895 N s/m ²

Kinematic viscosity

1 ft ² /s	=	0.09290 m ² /s
1 in ² /s	=	645.2 mm ² /s
1 cSt	=	1 mm ² /s

Electrical units

The conversion factors which follow are from the C.G.S. system to the SI system. (Note: in the C.G.S. system 1 e.m.u. = 3×10^{10} e.s.u. of charge).

capacitance	1 e.s.u.	= $\frac{1}{9} \times 10^{-11}$ F
charge	1 e.m.u.	= 10 C
current	1 e.m.u.	= 10 A
electric field strength	1 e.s.u.	= 3×10^4 V/m
electric flux density	1 e.s.u.	= $\frac{1}{12\pi} \times 10^{-5}$ C/m ²
electric polarisation	1 e.s.u.	= $\frac{1}{3} \times 10^{-5}$ C/m ²
inductance	1 e.m.u.	= 10^{-9} H
intensity of magnetisation	1 e.m.u.	= 10^3 A/m
magnetic field strength	1 e.m.u.	= $\frac{1}{4\pi} \times 10^3$ A/m
magnetic flux	1 e.m.u.	= 10^{-8} Wb
magnetic flux density	1 e.m.u.	= 10^{-4} Wb/m ²
magnetic moment	1 e.m.u.	= 10^{-3} A m ²
magnetomotive force	1 e.m.u.	= $\frac{10}{4\pi}$ A
mass susceptibility	1 e.m.u/g	= $4\pi \times 10^{-3}$ kg ⁻¹
potential	1 e.m.u.	= 10^{-8} V
resistance	1 e.m.u.	= 10^{-9} Ω

2. PHYSICAL CONSTANTS

Avogadro's number	N	= 6.023×10^{26} / (kg mol)
Bohr magneton	B	= 9.27×10^{-24} A m ²
Boltzmann's constant	k	= 1.380×10^{-23} J/K
Stefan-Boltzmann constant	σ	= 5.67×10^{-8} W/(m ² K ⁴)
characteristic impedance of Z ₀ free space		= $(\mu_0/\epsilon_0)^{\frac{1}{2}} = 120\pi\Omega$
electron volt	eV	= 1.602×10^{-19} J
electron charge	e	= 1.602×10^{-19} C
electronic rest mass	m _e	= 9.109×10^{-31} kg
electronic charge to mass ratio	e/m _e	= 1.759×10^{11} C/kg
Faraday constant	F	= 9.65×10^7 C/(kg mol)
permeability of free space	μ ₀	= $4\pi \times 10^{-7}$ H/m
permittivity of free space	ε ₀	= 8.85×10^{-12} F/m
Planck's constant	h	= 6.626×10^{-34} J s
proton mass	m _p	= 1.672×10^{-27} kg
proton to electron mass ratio	m _p /m _e	= 1836.1
standard gravitational acceleration	g	= 9.80665 m/s ² = 9.80665 N/kg
universal constant of gravitation	G	= 6.67×10^{-11} N m ² /kg ²
universal gas constant	R ₀	= 8.314 kJ/(kg mol K)
velocity of light in vacuo	c	= 2.9979×10^8 m/s
volume of 1 kg mol of ideal gas at 1 atm, 0°C		= 22.41 m ³

Temperature

$${}^{\circ}\text{C} = \frac{5}{9}({}^{\circ}\text{F} - 32)$$

$$\text{K} = \frac{5}{9}({}^{\circ}\text{F} + 459.67) = \frac{5}{9}{}^{\circ}\text{R} = {}^{\circ}\text{C} + 273.15$$

3 . SUMMARY OF "BASIC"

This language contains the facilities provided in most versions of extended BASIC. Some instructions may vary somewhat from one system to another; however, equivalents should be available. This applies particularly to String Functions, Commands and Control Codes, and to items marked with a +. We suggest that you modify the SYSTEM DEPENDENT INSTRUCTIONS MARKED BY + to conform to your own system and add other instructions in the spaces provided.

Arithmetic Variable Names

numeric variables: e.g. A,X,B4,Z1

arithmetic array variables: e.g. S(4),A(I+1),N2(1,J),
C(1,B(1))

String Variable Names

character string variables: e.g. B\$

character string array variables: e.g. Z\$(4),NS(A,B)

N.B. \$ may be f on some terminals. Use the key 'shift 4'.

Arithmetic Operators

+ exponentiation e.g. 2+3 gives 8

- unary minus

* / multiplication, division

+ - addition, subtraction

Operations inside any given pair of brackets are performed before those outside. Subject to this, BASIC performs operations in the order of the operators above. The only (+) exception is A+B, interpreted as A+(-B). Operators of equal priority are applied from left to right.

e.g. 2+(1+3/2*(1+1)) gives 16

Relational Operators (operate upon arithmetic and string values)

= >

< >=

<= (less than equal to) >< (not equal to)

Logical Operators

AND

XOR exclusive

OR inclusive

Matrix Operators

- + - addition or subtraction of matrices of equal dimensions
- * multiplication of conformable matrices
- * multiplication of a matrix by a scalar
e.g. MAT A = (K)*A

Arithmetic Functions (x represents any expression)

- | | |
|----------------------|--|
| PI | has the constant value 3.1415927 |
| SIN(x),COS(x),TAN(x) | sine, cosine, tangent (x in radians) |
| ATN(x) | arctan (radians) |
| LOG(x),LOG10(x) | natural log, common log |
| EXP(x) | exponentiation e^x where $e = 2.71828$ |
| SQR(x) | square root |
| SGN(x) | sign of x
(+ve gives 1, 0 gives 0, -ve gives -1) |
| ABS(x) | absolute value of x
(x) |
| INT(x) | largest integer $\leq x$ |
| RND or RND(x) | returns a random number between 0 and 1.
x, if present, is ignored. |

+ String Functions

- | | |
|------------------|--|
| LEN(A\$) | returns the number of characters in the string A\$, including trailing blanks |
| SUB\$(A\$,N1,N2) | creates a sub string from the string A\$ starting with the N1th character and N2 characters long |
| SUB\$(A\$,N) | creates a sub string from the string A\$, starting with the Nth character to the last character in A\$ |
| *CHR\$(x) | returns a one character string having the ASCII value x |
| *ASCII(A\$) | returns the ASCII value of the first character in A\$ |
| NUM\$(N) | creates the string of characters that would be printed by PRINT N; |
| NUM\$(N,field) | creates the string of characters that would be printed by PRINT USING "field",N; |
| VAL(A\$) | computes the value that would be generated by the INPUT of the characters of A\$ to an arithmetic variable |

*The ASCII value is based on seven bit characters. Treatment of the parity bit is system dependent.

+Error functions (only valid in an error handling routine entered by ONERROR)

ERR contains the error number of the most recent error
ERL contains the line number of the most recent error

Matrix functions

MAT Y = TRN(X) Y becomes the transpose of X
MAT Y = INV(X) Y becomes the inverse of X
DET contains the determinant of X after the evaluation of INV(X)

User defined functions - see DEF statement

Statements

Note - a program line may contain several statements separated by the colon (:) character

Type	Example
CLOSE	CLOSE 2
DATA	DATA 4.3,85,"MONDAY"
DEF	DEF FNA(X) = X+X DEF FN(A,B) = SQR(A+B) DEF FNF(M) IF M = 1 THEN FNF = .1 ELSE FNF = M*FNF(M-1) FNEND
DIM	DIM A(10),B\$(5,10)
END	END must be the last statement of a program
FOR	FOR X = 1 TO 10 FOR N = A TO A+R FOR I = 2 to 40 STEP 2
GOSUB	GOSUB 200
GOTO	GOTO 151
IF	IF B = A THEN 21 IF A > Z THEN PRINT "BIGGER" IF R < N+1 THEN R = N ELSE R = N+2 IF A > B OR B < C THEN STOP IF FNA(R) = B GOTO 200
INPUT	INPUT A INPUT "TYPE YOUR NAME",A\$ INPUT #4,N,M
LET	LET A = 20 LET A,B,C = 0 A\$ = "TEXT" (LET is optional)

MAT

MAT C = CON
all elements of C = 1
MAT B = IDN(10,10)
identity matrix
MAT A = ZER
all elements of A = 0
MAT B = ZER(5,10)
redimensions and zeros B
MATINPUT A,B,C(4)
MATINPUT # 3,A,C

MATPRINT

MATPRINT B
MATPRINT B(10,5);
MATPRINT # 2,A

MATREAD

MATREAD A,B(4,4)

NEXT

NEXT I

ON ERROR GOTO

ON ERROR GOTO 140

ON GOSUB

ON X GOSUB 200,250,300
ON FNA(A) + FNB(A) GOSUB 10,15,30,5

ON GOTO

ON A + 1 GOTO 14,25,50

OPEN

+

PRINT

PRINT A,B
PRINT "RESULT": X1
PRINT # 4, I*A,. "EXPERIMENT"; N

+ PRINT USING

PRINT USING "# # #", A,B
PRINT # 3, USING B\$,C,Z\$
PRINT USING 1000,X

RANDOM

RANDOM

READ

READ A,B\$,F1,C

REM

REMARK THIS IS A COMMENT

!

An exclamation mark at the beginning of a line is equivalent to REM
An exclamation mark after any statement causes the rest of the line to be treated as comment
X = !: ZERO CONTROL

RESTORE

RESTORE

RESUME

RESUME 240

RETURN

RETURN

STOP

STOP

TRACE

TRACE

+ %

PRINTUSING image, e.g. 1000% X = # #.#

+Commands

RUN	runs the current program
LOAD	loads a program from paper tape (or other medium)
CLEAR }	remove any existing program
NEW }	
LIST	LIST prints the current program LIST n prints line n LIST n-m prints lines n to m
DELETE	DELETE 40-45 deletes specified lines from the current program
SAVE	saves a program on paper tape (or other medium)
REP	edits a program line. Any non-numerical character can be used as separator e.g. REP10/X1/A REP30/,B/
RESEQUENCE	renumbers part or all of a program (including GOTO etc references) e.g. RESEQUENCE whole program, steps of 10 RESEQUENCE 900,1000 after old line 900, which becomes 1000, steps of 10 RESEQUENCE,,5 whole program, steps of 5

+Special control codes

ESC }	breaks into the program and stops it, typing BASIC READY
CR }	terminate a line of input
RETURN }	
\ or \$	(on the same key as L, not 4) Abandon the current line of input
+ or RUBOUT	delete the previous character or space (may be used repeatedly)

4. ANALYSIS

4.1 Vector algebra

$$\hat{a} = \underline{a} / |\underline{a}|$$

$$\underline{a} = a_1 \underline{i} + a_2 \underline{j} + a_3 \underline{k} \equiv (a_1, a_2, a_3)$$

$$a \equiv |\underline{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\underline{a} + \underline{b} = (a_1+b_1, a_2+b_2, a_3+b_3)$$

Scalar (dot) product:

$$\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = ab \cos \theta$$

Vector (cross) product:

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = ab \sin \theta \hat{n}$$

where $\hat{n} \perp \underline{a}, \hat{n} \perp \underline{b}$

Triple scalar product:

$$[\underline{a} \underline{b} \underline{c}] = \underline{a} \cdot \underline{b} \times \underline{c} = \underline{a} \times \underline{b} \cdot \underline{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Triple vector product:

$$\underline{a} \times (\underline{b} \times \underline{c}) = (a \cdot c) \underline{b} - (a \cdot b) \underline{c}$$

$$(\underline{a} \times \underline{b}) \times \underline{c} = (a \cdot c) \underline{b} - (b \cdot c) \underline{a}$$

Differentiation of vectors:

$$\frac{d}{dt}(\underline{a} + \underline{b}) = \frac{da}{dt} + \frac{db}{dt} \quad \frac{d}{dt}(f \underline{a}) = \frac{df}{dt} \underline{a} + f \frac{da}{dt}$$

$$\frac{d}{dt}(\underline{a} \cdot \underline{b}) = \underline{a} \cdot \frac{db}{dt} + \frac{da}{dt} \cdot \underline{b} \quad \frac{d}{dt}(\underline{a} \times \underline{b}) = \underline{a} \times \frac{db}{dt} + \frac{da}{dt} \times \underline{b}$$

$$\frac{d}{dt}(\underline{a} \cdot \underline{b} \times \underline{c}) = \frac{da}{dt} \cdot \underline{b} \times \underline{c} + \underline{a} \cdot \frac{db}{dt} \times \underline{c} + \underline{a} \cdot \underline{b} \times \frac{dc}{dt}$$

Gradient:

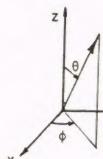
$$\text{grad } V = \underline{\nabla}V = \underline{i} \frac{\partial V}{\partial x} + \underline{j} \frac{\partial V}{\partial y} + \underline{k} \frac{\partial V}{\partial z} \text{ (Cartesian)}$$

$$= \underline{u_r} \frac{\partial V}{\partial r} + \underline{u_\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \underline{u_z} \frac{\partial V}{\partial z} \text{ (Cylindrical)}$$

$$\text{where } \underline{u_r} = \underline{i} \cos\phi + \underline{j} \sin\phi$$

$$\underline{u_\phi} = -\underline{j} \sin\phi + \underline{i} \cos\phi$$

$$\underline{u_z} = \underline{k}$$



$$= \underline{u_r} \frac{\partial V}{\partial r} + \underline{u_\theta} \frac{\partial V}{\partial \theta} + \frac{\underline{u_\phi}}{r \sin\theta} \frac{\partial V}{\partial \phi} \text{ (Spherical)}$$

$$\text{where } \underline{u_r} = \underline{i} \cos\phi \sin\theta + \underline{j} \sin\phi \sin\theta + \underline{k} \cos\theta$$

$$\underline{u_\theta} = \underline{i} \cos\phi \cos\theta + \underline{j} \sin\phi \cos\theta - \underline{k} \sin\theta$$

$$\underline{u_\phi} = -\underline{j} \sin\phi + \underline{i} \cos\phi$$

Divergence:

$$\text{div } \underline{F} = \underline{\nabla} \cdot \underline{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \text{ (Cartesian)}$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (r F_r) + \frac{1}{r} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \text{ (Cylindrical)}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (F_\theta \sin\theta) + \frac{1}{r \sin\theta} \frac{\partial F_\phi}{\partial \phi} \text{ (Spherical)}$$

Curl:

$$\text{curl } \underline{F} = \underline{\nabla} \times \underline{F} = \underline{i} \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \underline{j} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \underline{k} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \text{ (Cartesian)}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \frac{1}{r} \begin{vmatrix} \underline{u_r} & \underline{r u_\phi} & \underline{u_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & r F_\phi & F_z \end{vmatrix} \text{ (Cylindrical)}$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \underline{u_r} & \underline{r u_\theta} & \underline{r \sin\theta u_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & r F_\theta & r \sin\theta F_\phi \end{vmatrix} \text{ (Spherical)}$$

Laplace:

$$\underline{\nabla} \cdot \underline{\nabla} V = \underline{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \text{ (Cartesian)}$$

$$= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \text{ (Cylindrical)}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

(Spherical)

Space curves:

$$\underline{v} = \underline{u} \frac{ds}{dt}, \quad s = \text{arc length} \quad \underline{u} = \text{unit tangent}$$

$$\underline{a} = \frac{\underline{v}^2}{\rho} \underline{n} + \frac{d\underline{v}}{dt} \underline{u} \quad \underline{n} = \text{unit 'inward' normal}$$

$$\frac{d\underline{u}}{ds} = \frac{1}{\rho} \underline{n} \quad \rho = \text{radius of curvature}$$

$$\underline{b} = \underline{u} \times \underline{n}, \quad \underline{b} = \text{binormal vector}$$

$$\frac{d\underline{b}}{ds} = -\frac{1}{\tau} \underline{n}, \quad \frac{d\underline{n}}{ds} = \frac{1}{\tau} \underline{b} - \frac{1}{\rho} \underline{u}, \quad \frac{1}{\tau} = \text{torsion}$$

Identities:

$$\underline{\nabla} \cdot \underline{\phi u} = \underline{\phi} \underline{\nabla} \cdot \underline{u} + \underline{u} \cdot \underline{\nabla} \phi$$

$$\underline{\nabla} x \phi \underline{u} = \phi \underline{\nabla} x \underline{u} + \underline{\nabla} x \phi \underline{u}_0$$

$$\underline{\nabla} \cdot \underline{u} \underline{x} \underline{v} = \underline{v} \cdot \underline{\nabla} x \underline{u} - \underline{u} \cdot \underline{\nabla} x \underline{v}$$

4.2 Series

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots,$$

for arbitrary α , $|x| < 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n}{(2n)!} x^{2n} + \dots \text{ for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n}{(2n+1)!} x^{2n+1} + \dots \text{ for all } x$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \text{ for } |x| < \pi/2$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^n}{(n+1)} x^{n+1} + \dots$$

for $-1 < x < 1$

Taylor's

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \dots$$

$$+ \frac{h^{n-1}}{(n-1)!} f^{(n-1)}(a) + \frac{h^n}{n!} f^{(n)}(c) \text{ where } a < c < a+h$$

Maclaurin's

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \dots$$

$$+ \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + \frac{x^n}{n!} f^{(n)}(\theta x) \text{ where } 0 < \theta < 1$$

Stirling's formula for n!

For n large, $n! \approx \sqrt{(2\pi)} n^{n+\frac{1}{2}} e^{-n}$

or, $\log_{10} n! \approx 0.39909 + (n+\frac{1}{2}) \log_{10} n - 0.43429n$.

Fourier series

(i) General formulae

If $f(x)$ is periodic of period $2L$, $f(x+2L) = f(x)$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

where

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, 3, \dots$$

If $f(x)$ is an even function of x , i.e., $f(-x) = f(x)$

$$\text{then } a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx \quad n = 0, 1, 2, \dots$$

and $b_n = 0 \quad n = 1, 2, 3, \dots$

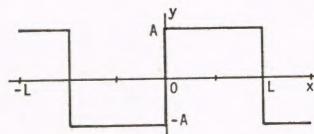
If $f(x)$ is an odd function of x , i.e., $f(-x) = -f(x)$

$$\text{then } a_n = 0 \quad n = 0, 1, 2, \dots$$

and $b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, 3, \dots$

(ii) Special waveforms, all of period $2L$

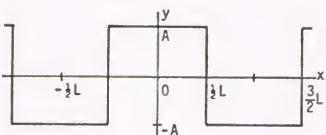
(a) Square wave, sine series



$$f(x) = \frac{4A}{\pi} \left[\sin \frac{\pi x}{L} + \frac{1}{3} \sin \frac{3\pi x}{L} + \frac{1}{5} \sin \frac{5\pi x}{L} + \dots \right]$$

$$\text{mean square value} = A^2$$

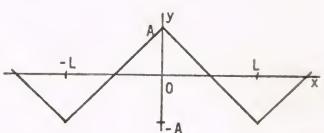
(b) Square wave, cosine series



$$f(x) = \frac{4A}{\pi} \left[\cos \frac{\pi x}{L} - \frac{1}{3} \cos \frac{3\pi x}{L} + \frac{1}{5} \cos \frac{5\pi x}{L} - \dots \right]$$

$$\text{mean square value} = A^2$$

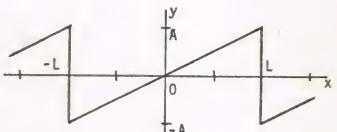
(c) Triangular wave



$$f(x) = \frac{8A}{\pi^2} \left[\cos \frac{\pi x}{L} + \frac{1}{3^2} \cos \frac{3\pi x}{L} + \frac{1}{5^2} \cos \frac{5\pi x}{L} + \dots \right]$$

$$\text{mean square value} = \frac{A^2}{3}$$

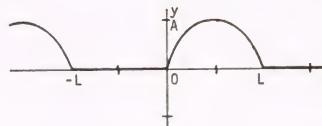
(d) Saw-tooth wave



$$f(x) = \frac{2A}{\pi} \left[\sin \frac{\pi x}{L} - \frac{1}{2} \sin \frac{2\pi x}{L} + \frac{1}{3} \sin \frac{3\pi x}{L} - \dots \right]$$

$$\text{mean square value} = \frac{A^2}{3}$$

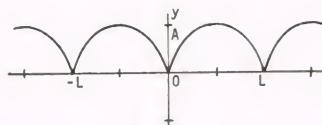
(e) Half-wave rectification



$$f(x) = \frac{A}{2} \sin \frac{\pi x}{L} + \frac{2A}{\pi} \left[\frac{1}{2} - \frac{1}{3} \cos \frac{2\pi x}{L} - \frac{1}{15} \cos \frac{4\pi x}{L} - \dots \right]$$

$$\text{mean square value} = \frac{A^2}{4} \quad \text{average value} = \frac{A}{\pi}$$

(f) Full-wave rectification



$$f(x) = \frac{4A}{\pi} \left[\frac{1}{2} - \frac{1}{3} \cos \frac{2\pi x}{L} - \frac{1}{15} \cos \frac{4\pi x}{L} - \dots \right]$$

$$\text{mean square value} = \frac{A^2}{2} \quad \text{average value} = \frac{2A}{\pi}$$

4.3 Trigonometric, hyperbolic and algebraic relations

$$2 \sin A \cos B = \sin(A-B) + \sin(A+B)$$

$$2 \cos A \cos B = \cos(A-B) + \cos(A+B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin^2 A + \cos^2 A = 1$$

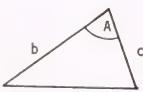
$$\sec^2 A = \tan^2 A + 1$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{sin } A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

$$= \frac{2}{bc} \text{ area}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$



Relation for Spherical Triangles

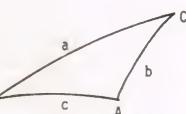
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = -\cos b \cos c + \sin b \sin c \cos a$$

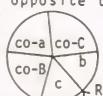
$$\frac{\sin \frac{a}{2}}{\sin \frac{A}{2}} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}} \text{ where } s = \frac{1}{2}(a+b+c)$$

$$\frac{\sin \frac{a}{2}}{\sin \frac{A}{2}} = \sqrt{\frac{\cos s \cos(S-A)}{\sin B \sin C}} \text{ where } S = \frac{1}{2}(A+B+C)$$



Napier's Rules for right spherical triangles:

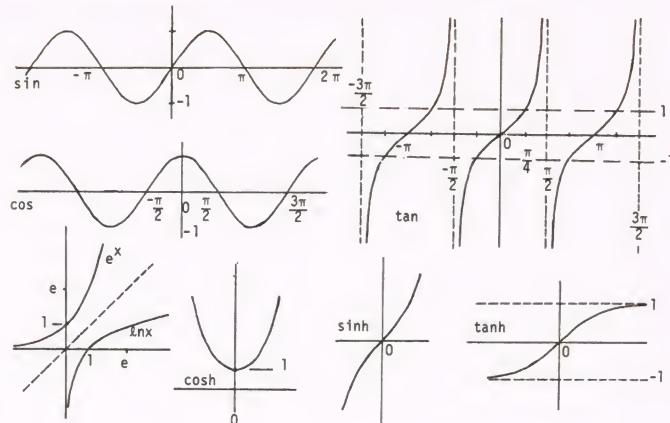
Arrange the five parts about the right angle with 'co' attached to the three parts opposite the right angle. E.g. for the right angle at A we have



N.B. co-a is the complement of a.
i.e. $90^\circ - a$

Then: The sine of the middle part is the product of the tangents of adjacent parts and is the product of the cosines of opposite parts.

N.B. A leg and its opposite angle are always in the same quadrant. If the hypotenuse is less than 90° the legs are in the same quadrant, otherwise they are in opposite quadrants.



$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cos iz = \cosh z$$

$$\cosh iz = \cos z$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sin iz = i \sinh z$$

$$\sinh iz = i \sin z$$

$$e^z = \cosh z + i \sinh z$$

$$\log_{10}(10^x) \equiv \log_{10}(\text{antilog}_{10}x) \equiv x \equiv 10^{\log_{10}x} \equiv e^{\log_e x} = e^{izx}$$

$$a^2 - b^2 = (a+b)(a-b) : a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

equations of curves

circle

$$x^2 + y^2 = a^2$$

ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parabola

$$y^2 = ax$$

4.4 Complex numbers

$$z = r(\cos\theta + i\sin\theta) = x + iy$$

$$= r e^{i(\theta+2n\pi)} \quad (n = 0, \pm 1, \pm 2, \dots)$$

$$e^{iz} = \cos z + i \sin z \quad [\text{Euler's Formula}]$$

$$x + iy = \sqrt{x^2 + y^2} e^{i \tan^{-1}(y/x)}; \quad z^c = e^{c \ln z}$$

N.B. $\tan^{-1}(y/x)$ must be chosen to lie in the appropriate quadrant

4.5 Partial differentiation

(a) If $F = f(x,y)$, where $x = X(t)$, $y = Y(t)$ then

$$F = F(t) \text{ and } \frac{dF}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

(b) If $F = f(x,y)$, where $y = Y(x)$, then $F = F(x)$ and

$$\frac{dF}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx}$$

(c) If $F = f(x,y)$, where $x = X(u,v)$, $y = Y(u,v)$ then

$$F = F(u,v) \text{ and } \frac{\partial F}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u},$$

$$\frac{\partial F}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v}.$$

4.6 Differential Equations

(i) First Order

Type	Characteristic	Method of solution
separable	$y' = P(x)Q(y)$	rearrange:- $\int \frac{1}{Q} dy = \int P dx + c$
homogeneous	$y' = f\left(\frac{y}{x}\right)$	by substitution $y = ux$ to make equation separable
exact	$M(x,y)dx + N(x,y)dy$ where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	$\frac{\partial G}{\partial x} = M, \quad \frac{\partial G}{\partial y} = N$ Solve for G
linear	$y' + P(x)y = Q(x)$	multiply through by $e^{\int P dx}$

(ii) Second Order, linear with constant coefficients

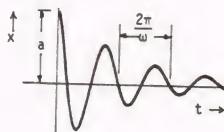
$$m\ddot{x} + \alpha\dot{x} + kx = 0$$

$$\ddot{x} + 2\xi\omega_0 \dot{x} + \omega_0^2 x = 0, \quad \xi = \frac{\alpha}{2\sqrt{mk}}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

(a) $\xi < 1$ (underdamping)

$$x = a e^{-\xi\omega_0 t} \cos(\omega t - \theta)$$

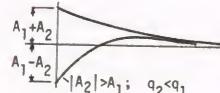
$$\omega = \omega_0 \sqrt{1-\xi^2}$$



(b) $\xi > 1$ (overdamping)

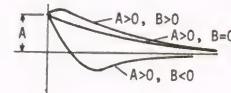
$$x = A_1 e^{-q_1 t} + A_2 e^{-q_2 t}$$

$$\text{where } q_1, q_2 = \omega_0 \{\xi \pm \sqrt{(\xi^2 - 1)}\}$$



(c) $\xi = 1$ (critical damping)

$$x = (A + Bt)e^{-\xi\omega_0 t}$$



Forced oscillations

$$\ddot{x} + 2\xi\omega_0 \dot{x} + \omega_0^2 x = a \cos pt, \quad a = \frac{F}{m}, \quad x_1 = \frac{F}{K}$$

$$x = \frac{Aa \cos(pt - \phi)}{\omega_0^2}$$

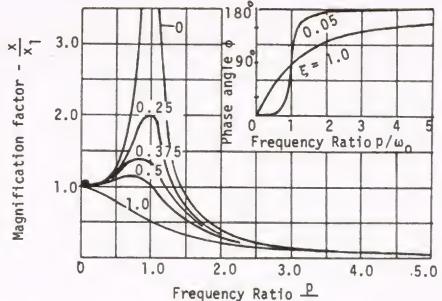
$$\tan \phi = \frac{2\xi p / \omega_0}{1 - \left(\frac{p}{\omega_0}\right)^2}$$

$$A = \left| \frac{x}{x_1} \right| = \frac{1}{\left[\left(1 - \left(\frac{p}{\omega_0} \right)^2 \right)^2 + \left(2\xi \frac{p}{\omega_0} \right)^2 \right]^{\frac{1}{2}}}$$

At resonance $p = \omega_0 \sqrt{1 - 2\xi^2}$

$$x = \frac{x_1}{2\xi\sqrt{1-\xi^2}}$$

$$\tan \phi = \frac{\sqrt{1-2\xi^2}}{\xi}$$



4.7 Rules of Differentiation and Integration

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{1}{v^2} \left(v \frac{du}{dx} - u \frac{dv}{dx}\right)$$

$$\int uv \, dx = uw - \int \frac{du}{dx} w \, dx, \text{ where } w = \int v \, dx$$

4.8 Standard Differentials and Integrals

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\frac{d}{dx} e^{ax} = ae^{ax}$$

$$\int e^{ax} \, dx = \frac{e^x}{a}, \quad a \neq 0$$

$$\frac{d}{dx} a^x = a^x \ln a$$

$$\int a^x \, dx = \frac{a^x}{\ln a}, \quad a > 0, \quad a \neq 1$$

$$\frac{d}{dx} x^x = x^x(1+\ln x)$$

$$\int \ln x \, dx = x(\ln x - 1)$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\int \cos x \, dx = \sin x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\int \sin x \, dx = -\cos x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \sec^2 x \, dx = \tan x$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x, \quad |x| < 1$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\int \sinh x \, dx = \cosh x$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\int \cosh x \, dx = \sinh x$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x$$

$$\frac{d}{dx} \coth x = -\operatorname{cosech}^2 x$$

$$\int \operatorname{cosech}^2 x \, dx = -\coth x$$

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x$$

$$= \ln |x + \sqrt{1+x^2}|$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$$

$$= \ln |x + \sqrt{(x^2-1)}|, x \geq 1$$

$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1} x$$

$$= \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|, \quad x^2 < 1$$

$$\frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$$

$$\int \frac{dx}{x^2-1} = -\coth^{-1} x$$

$$= \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right|, \quad x^2 > 1$$

Some definite integrals (m, n integers)

$$\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx = \begin{cases} \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{3}{4} \frac{1}{2} \frac{\pi}{2}, & n \text{ even} \\ \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{4}{5} \frac{2}{3} 1, & n \text{ odd} \end{cases}$$

$$I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx = \binom{m-1}{m+n} I_{m-2,n} = \binom{n-1}{m+n} I_{m,n-2}, \quad m \neq -n$$

$$\int_0^{\pi} \sin mx \sin nx dx = \int_0^{\pi} \cos mx \cos nx dx = 0 \quad (m \neq n)$$

$$\int_0^{\pi} \sin nx \cos mx dx = 0$$

$$\int_0^{\infty} e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}, \quad a > 0$$

$$\int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}, \quad a > 0$$

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

The error function $\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du$ (refer to page 30 for tabulated value)

$$\int_0^{\pi} \frac{\sin \theta \cos \theta}{(1+\varepsilon \cos \theta)^3} d\theta = \frac{-2\varepsilon}{(1-\varepsilon^2)^2}$$

$$\int_0^{\pi} \frac{\sin^2 \theta d\theta}{(1+\varepsilon \cos \theta)^3} = \frac{\pi}{2(1-\varepsilon^2)^{3/2}}$$

$$\int_0^{2\pi} \frac{d\theta}{(1+\varepsilon \cos \theta)} = \frac{2\pi}{(1-\varepsilon^2)^{1/2}}$$

4.9 Laplace Transforms

$$\text{Definition} \quad F(s) = L[f(t)] = \int_0^{\infty} f(t)e^{-st} dt$$

Theorems

$$\text{Linearity} \quad L[af(t)+bg(t)] = aF(s)+bG(s)$$

$$\text{Final Value} \quad \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

$$\text{Initial Value} \quad \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$$

$$\text{Differentiation} \quad L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$$

$$L\left[\frac{d^2f(t)}{dt^2}\right] = s^2F(s) - sf(0) - f'(0)$$

$$\text{Integration} \quad L\left[\int f(t) dt\right] = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$$

$$\text{First Shifting} \quad L[e^{at}f(t)] = F(s-a)$$

$$\text{Second Shifting} \quad L[f(t-a)] \quad t > a = e^{-as}F(s)$$

$$\text{Convolution} \quad L[f*g] \equiv L\left[\int_0^t f(u)g(t-u)du\right] = F(s)G(s)$$

$$\text{Partial Differentiation} \quad L\left[\frac{\partial f(t,a)}{\partial a}\right] = \frac{\partial}{\partial a} F(s,a)$$

$$\text{Time Multiplication} \quad L[tf(t)] = -\frac{dF(s)}{ds}$$

Transform Pairs

Function

$$1$$

$$H(t-T) = \begin{cases} 0 & t < T \\ 1 & t \geq T \end{cases}$$

Laplace Transform

$$\frac{1}{s}$$

$$\frac{1}{s} e^{-sT}$$

$$t^n$$

$$\frac{n!}{s^{n+1}}$$

$$e^{-at}$$

$$\frac{1}{s+a}$$

$$\sin \omega t$$

$$\frac{\omega}{s^2 + \omega^2}$$

$$\cos \omega t$$

$$\frac{s}{s^2 + \omega^2}$$

$$1 - e^{-t/T}$$

$$\frac{1}{s(1+s)}$$

$$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin(\omega_n \sqrt{1-\xi^2} t)$$

$$\frac{1}{1 + 2\frac{\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

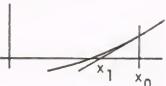
$$1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \left[\omega_n \sqrt{1-\xi^2} t + \cos^{-1} \xi \right] = \frac{1}{s \left(1 + 2 \frac{\xi s}{\omega_n} + \frac{s^2}{\omega_n^2} \right)}$$

4.10 Numerical analysis

(i) Approximate solution of an algebraic equation $f(x) = 0$

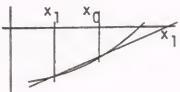
(a) Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



(b) Secant Method

$$x_1 = \frac{-x_0 f(x_{-1}) + x_{-1} f(x_0)}{f(x_0) - f(x_{-1})}$$



(ii) Least-squares fitting of a straight line

If y_i ($i = 1, 2, \dots, n$) are the experimentally observed values of y at chosen (exact) values of x_i of the variable x , the line of 'best fit' passes through the centroid

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

and is given by $y = mx + c$ where,

$$m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}, \quad c = \bar{y} - m\bar{x}$$

$$= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

(iii) Finite-difference formulae

$$\Delta f(x) = f(x+h) - f(x)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2)$$

$$f'''(x) = \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3}$$

(iv) Lagrange's interpolation formula for unequal intervals.

The polynomial $P(x)$ of degree 2 passing through the three points (x_i, y_i) , $i = 1, 2, 3$, is

$$P(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3$$

(v) Formulae for numerical integration

Equal intervals h

$$x_n = x_0 + nh, \quad y_n = y(x_n)$$

(a) Trapezoidal Rule (1-strip):

$$\int_{x_0}^{x_1} y(x) dx = \frac{h}{2}[y_0 + y_1] + \epsilon, \\ \epsilon \approx -\frac{h^3}{12} y'' \text{ or, } -\frac{h}{12} \Delta^2 y_0$$

(b) Simpson's Rule (2-strip):

$$\int_{x_0}^{x_2} y(x) dx = \frac{h}{3}[y_0 + 4y_1 + y_2] + \epsilon, \\ \epsilon \approx -\frac{h^5}{90} y_1^{(4)}, \text{ or } -\frac{h}{90} \Delta^4 y_0$$

(vi) Runge-Kutta

$$\text{2nd order: } y_{n+1} = y_n + \frac{h}{2} \left\{ f(x_n, y_n) + f(x_n + h, y_n + k_1) \right\}$$

$$\text{4th order: } y_{n+1} = y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf \left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2} \right)$$

$$k_3 = hf \left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2} \right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

5. ANALYSIS OF EXPERIMENTAL DATA

5.1 Probability distributions for discrete random variables

Notation: $P(r) = f(r) \Rightarrow$ the probability distribution of random variable r is $f(r)$

$$\mu = \text{mean value of } r = \sum_{i=1}^N r_i f(r_i)$$

$$\sigma^2 = \text{variance of } r = \sum_{i=1}^N r_i^2 f(r_i) - \mu^2$$

$$\binom{n}{r} = \text{binomial coefficient} = \frac{n!}{(n-r)!r!} = \binom{n}{n-r}$$

evaluate using Pascal's Triangle

$r = 0$	1	2	3	4	5	6	7	8	9	10
$n = 0$	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	126	84	36	9	1
10	1	10	45	120	210	252	210	120	45	10

(a) Binomial:

n = number of trials with constant probability p of success in each

r = number of successes

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r} \quad r = 0, 1, 2, \dots n$$

$$\mu = np, \quad \sigma^2 = np(1-p)$$

(b) Poisson:

μ = mean rate of occurrence of an event

r = number of events actually occurring in unit time

$$P(r) = e^{-\mu} \mu^r / r! \quad r = 0, 1, \dots$$

$$\sigma^2 = \mu$$

5.2 Probability distributions for continuous random variables

(a) Exponential:

$$\text{probability density function } f(x) = \lambda e^{-\lambda x},$$

$$x > 0, \lambda > 0$$

$$\mu = 1/\lambda$$

$$\sigma^2 = 1/\lambda^2$$

(b) Normal: the standardised normal distribution, $N(0,1)$

has probability density function

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

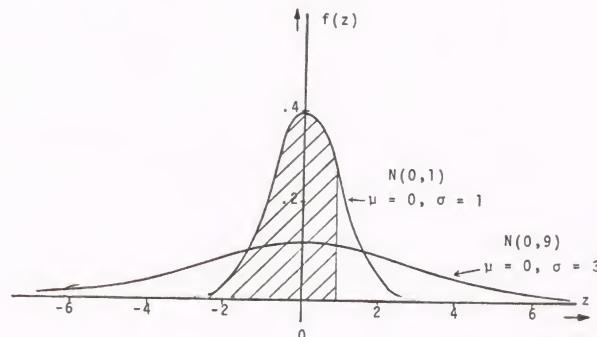
$$\mu = 0$$

$$\sigma = 1$$

Φ = cumulative distribution function

$\Phi(z) = \text{probability that the random variable is observed to have a value } \leq z \text{ (the shaded area shown)}$

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$$



For negative z use $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.0	.5000	1.0	.8413	2.0	.9772
.1	.5398	.1	.8643	.1	.9821
.2	.5793	.2	.8849	.2	.9861
.3	.6179	.3	.9032	.3	.9893
.4	.6554	.4	.9192	.4	.9918
.5	.6915	1.5	.9332	2.5	.9938
.6	.7257	.6	.9452	.6	.9953
.7	.7580	.7	.9554	.7	.9965
.8	.7881	.8	.9641	.8	.9974
.9	.8159	.9	.9713	.9	.9981
				3.0	.9987
				4.0	.99997

Percentage points of the Normal Distribution $N(0,1)$

$\Phi(z)$	% (1-tail)	% (2-tails)	z
.9500	5.0	10	1.6449
.9750	2.5	5	1.9600
.9900	1.0	2	2.3263
.9950	0.5	1	2.5758

The general normal distribution $N(\mu, \sigma^2)$ has probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

$$\text{where } \int_{-\infty}^{\infty} f(x) dx = 1$$

and cumulative distribution function $F(x)$

$$F(x) = \int_{-\infty}^{(x-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

To use tables of $\Phi(z)$, take $z = \frac{x-\mu}{\sigma}$.

5.3 Experimental Samples

x_1, x_2, \dots, x_n denote a set of n observations of a random variable having a normal distribution whose population mean μ is unknown.

$$\text{Range} = x_{\max} - x_{\min}$$

$$\text{Sample mean } \bar{x} = \frac{1}{n} \sum x_i$$

$$\text{Average deviation} = \frac{1}{n} \sum |x_i - \bar{x}|$$

$$\text{Sample standard deviation} = s,$$

$$\text{Sample variance} = s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$\text{Distribution of } \bar{x} \text{ is } N(\mu, \sigma^2/n)$$

$$\text{Distribution of } s \text{ is } N(\sigma, \sigma^2/(n-1))$$

$$\text{Distribution of } \frac{\bar{x} - \mu}{s/\sqrt{n}} \text{ is } N(0,1)$$

$$\text{i.e. standard error of sample means} = \frac{\sigma}{\sqrt{n}}$$

If population variance σ^2 is known,

$$95\% \text{ confidence interval for } \mu \text{ is } \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$99\% \text{ " " " " " is } \bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$$

If population variance σ^2 is unknown: $\frac{\bar{x} - \mu}{s/\sqrt{n}}$ has the t-distribution

with $n-1$ degrees of freedom (t_{n-1}) and the 95% confidence interval for μ is obtained from $\bar{x} \pm t_c s/\sqrt{n}$ and the table.

95% points of the t-distribution

$n-1$	t_c	$n-1$	t_c	$n-1$	t_c
1	12.7	6	2.45	12	2.18
2	4.30	7	2.36	15	2.13
3	3.18	8	2.31	20	2.09
4	2.78	9	2.26	30	2.04
5	2.57	10	2.23	60	2.00
				∞	1.96

Thus for $n > 20$, $\bar{x} \pm 1.96 s/\sqrt{n}$ is a good approximation to the population mean with a 95% confidence.

5.4 Combination of Errors

If results are Normally Distributed, the Most Probable Error S_z in the calculated result $z = f(x, y, \text{etc.})$, due to the independent standard errors $S_x, S_y, \text{etc.}$ in $x, y, \text{etc.}$ is given by,

$$(S_z)^2 = \left(\frac{\partial z}{\partial x} S_x \right)^2 + \left(\frac{\partial z}{\partial y} S_y \right)^2 + \dots \text{etc.}$$

If the function f consists of multiplied and divided terms ONLY (i.e. no addition or subtraction)

$$\left(\frac{S_z}{z} \right)^2 = \left[n \frac{S_x}{x} \right]^2 + \left[m \frac{S_y}{y} \right]^2 + \dots \text{etc.}$$

where $n, m, \text{etc.}$ are the powers of $x, y, \text{etc.}$ in f .

Notes

- (1) The Maximum Possible Error ($\delta z = \frac{\partial z}{\partial x} \delta x + \frac{\partial z}{\partial y} \delta y, \text{etc.}$) is rarely of interest in engineering
- (2) Instrument 'rounding off' error $\pm \delta x$ may be treated as a Normally Distributed error by the equivalence $S_x \cong \frac{2}{3} \delta x$.

6. MECHANICS

Moments of inertia and Second moments of area - General theorems

N.B. The symbol I is used for both second moment of area and moment of inertia.

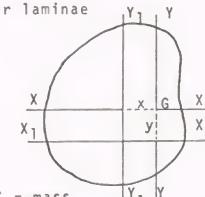
- (i) Parallel axis theorem: Solids or laminae

Centroid is at G

Centre of mass is at G

$$I_{x_1 x_1} = I_{XX} + Cy^2$$

$$I_{y_1 y_1} = I_{YY} + Cx^2$$



where for moment of inertia $C = \text{mass}$
and for second moment of area of a lamina $C = \text{area}$

- (ii) Perpendicular axis theorem for laminae:

$$\text{Polar second moment } J_o = I_{XX} + I_{YY} = I_{ZZ}$$

Radius of gyration k

$$\text{Second moment of area } I = Ak^2$$

$$\text{Moment of inertia } I = mk^2$$

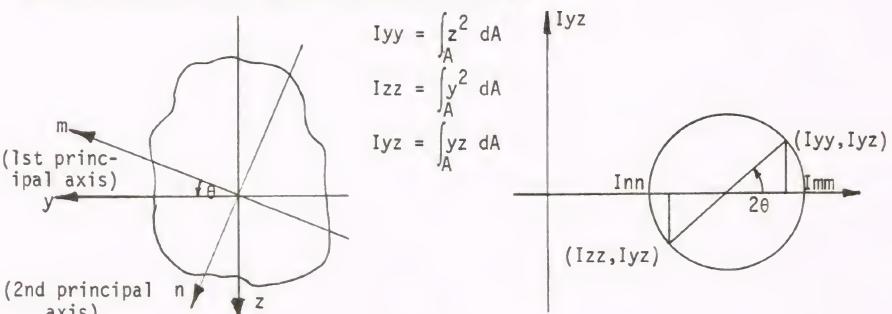
where $A = \text{area}, m = \text{mass.}$

(a) UNIFORM ROD	k_{XX}^2	k_{YY}^2	A
	-	$\frac{k^2}{12}$	-
(b) LAMINAE			
Rectangle	$\frac{l^2}{12} d^2$	$\frac{l^2}{12} b^2$	db
Circle	$\frac{1}{4} a^2$	$\frac{1}{4} a^2$	πa^2

(b) LAMINAE (cont)	k_{XX}^2	k_{YY}^2	A
Semi-circle	$a^2 \left \frac{1}{4} - \left(\frac{4}{3\pi} \right)^2 \right $	$\frac{1}{4} a^2$	$\frac{\pi a^2}{2}$
Triangle	$\frac{1}{18} h^2$ $k_{XY}^2 = \frac{1}{36} h(b_1 - b_2)$	$\frac{1}{18} (b_1^2 + b_1 b_2 + b_2^2)$	$\frac{h}{2}(b_1 + b_2)$
Ellipse	$\frac{b^2}{4}$	$\frac{a^2}{4}$	πab
(c) SOLIDS	k_{XX}^2	k_{YY}^2 and k_{ZZ}^2	V
Cylinder	$\frac{1}{2} a^2$	$\frac{1}{4} a^2 + \frac{1}{12} l^2$	$\pi a^2 l$
Thin-walled cylinder	$a^2 + \frac{1}{4} t^2$	$\frac{1}{2} a^2 + \frac{1}{8} t^2 + \frac{1}{12} l^2$	$2\pi a t l$
Thick walled cylinder	$\frac{1}{2} (R^2 + r^2)$	$\frac{l^2}{12} + \frac{R^2 + r^2}{4}$	$\pi (R^2 - r^2) l$

(c) SOLIDS (cont)	k_{XX}^2 and k_{ZZ}^2	k_{YY}^2	V
Sphere	$\frac{2}{5} a^2$	$\frac{2}{5} a^2$	$\frac{4}{3} \pi a^3$
Cone	$\frac{3(4a^2 + h^2)}{80}$	$\frac{3a^2}{10}$	$\frac{\pi}{3} a^2 h$

Mohr's Circle for Second Moment of Area



Constant acceleration equations

$$\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2ax \\ x &= ut + \frac{1}{2} at^2 \end{aligned}$$

Accelerations due to rotation

$$\text{Coriolis} = 2 \omega \times \left[\frac{\partial r}{\partial t} \right]$$

$$\text{Central} = \omega \times (\omega \times r)$$

Friction

coefficient of static friction $\mu = \tan \phi$
for no slipping $\frac{F}{N} \leq \mu$

DRY SLIDING FRICTION COEFFICIENTS

Clutches	0.3-0.4
Brakes (lining) " (pads)	0.35-0.5 ~0.3
Nylon/Steel	0.3-0.5
Filled PTFE/Steel	0.05-0.3
Perspex/Steel	~0.5
Rubber/Steel	0.6-0.9
Rubber/Asphalt	0.5-0.8
Lignum vitae/Steel	~0.1

7. PROPERTIES AND MECHANICS OF SOLIDS

7.1 Bonding

(a) Condon-Morse Equation $V_{\text{total}} = \frac{-Ae^2}{r^n} + \frac{B}{r^m} + C$

(b) Ionic Bond Equation $V_0 = \frac{-Z_1^* Z_2^* e^2}{4\pi\epsilon_0 r} \left(1 - \frac{1}{n}\right) + \Delta E$

(c) Theoretical Density $\rho = \frac{nA}{VN}$

7.2 Atomic sizes in substitutional alloys

Element	Seitz radius r_s (Å) (at 200°C)	Effective valency in solution
Al	1.58	3
Au	1.59	1
Cu	1.41	1
Fe(α)	1.41	?
Mg	1.85	2
Ni	1.38	1
P	1.58	3
Pb	1.95	4
Si	1.67	4
Sn	1.86	4
Zn	1.54	2

7.3 Phase Transformations

Length and volume changes may be related by:-

$$(1 + \Delta V/V) = (1 + \Delta L/L)^3$$

7.4 Crystallography

(a) In the Miller system:

Specific Plane (h, k, l)

Family of Planes $\{h, k, l\}$

Specific Direction $[h, k, l]$

Family of Directions $\langle h, k, l \rangle$

(b) Inter-planar spacings for Cubics

$$d_{(h, k, l)} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{a}{\sqrt{N}}$$

(c) Quadratic Forms of Miller Indices (N values)

Cubic Structure

N values

Simple	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20....
Face Centred	3, 4, 8, 11, 12, 16, 19, 20, 24, 27, 32....
Body Centred	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 30....
Diamond	3, 8, 11, 16, 19, 24, 27, 32....

7.5 Defects and Diffusion Data

(a) Number of Defects $n = \alpha N e^{-\frac{Q}{kT}}$

(b) Diffusivity $D = D_0 e^{-\frac{Q}{kT}}$

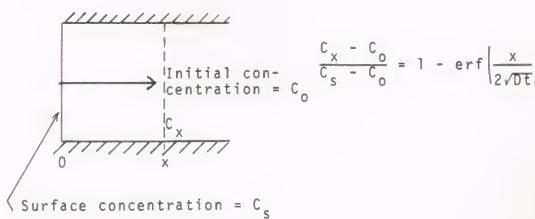
Note: these equations may be expressed in terms of R_0 rather than k , the value of Q must be quoted in the appropriate units.

(c) Macroscopic Diffusion

(i) D constant with composition, $\frac{dc}{dx}$ constant with time
 $J = -D \frac{dc}{dx}$ (This is a special case of (ii))

(ii) D constant with composition, $\frac{dc}{dx}$ varies with time
 $\frac{dc}{dt} = D \frac{d^2 c}{dx^2}$

Solution for a constant surface potential and impermeable sides



7.6 Selected Values of Error Function $\frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du$

z	$erf z$						
0.00	0.0000	0.68	0.6638	1.36	0.9456	2.00	0.9953
0.02	0.0226	0.70	0.6778	1.38	0.9490	2.05	0.9963
0.04	0.0451	0.72	0.6914	1.40	0.9523	2.10	0.9970
0.06	0.0676	0.74	0.7047	1.42	0.9554	2.15	0.9976
0.08	0.0901	0.76	0.7175	1.44	0.9583	2.20	0.9981
0.10	0.1125	0.78	0.7300	1.46	0.9611	2.25	0.9983
0.12	0.1348	0.80	0.7421	1.48	0.9637	2.30	0.9989
0.14	0.1570	0.82	0.7538	1.50	0.9661	2.35	0.9991
0.16	0.1790	0.84	0.7651	1.52	0.9684	2.40	0.9993
0.18	0.2009	0.86	0.7761	1.54	0.9706	2.45	0.9995
0.20	0.2227	0.88	0.7867	1.56	0.9726	2.50	0.9996
0.22	0.2443	0.90	0.7969	1.58	0.9746	2.55	0.9997
0.24	0.2657	0.92	0.8068	1.60	0.9764	2.60	0.9998
0.26	0.2869	0.94	0.8163	1.62	0.9780	2.65	0.9998
0.28	0.3079	0.96	0.8254	1.64	0.9796	2.70	0.9999
0.30	0.3286	0.98	0.8342	1.66	0.9811	2.75	0.9999
0.32	0.3491	1.00	0.8427	1.68	0.9825	2.80	0.9999
0.34	0.3694	1.02	0.8508	1.70	0.9838	2.85	0.9999
0.36	0.3893	1.04	0.8587	1.72	0.9850	2.90	1.0000
0.38	0.4090	1.06	0.8661	1.74	0.9861	2.95	1.0000
0.40	0.4284	1.08	0.8733	1.76	0.9872	3.00	1.0000
0.42	0.4475	1.10	0.8802	1.78	0.9882	4.00	1.0000
0.44	0.4662	1.12	0.8868	1.80	0.9891		
0.46	0.4847	1.14	0.8931	1.82	0.9899		
0.48	0.5028	1.16	0.8991	1.84	0.9907		
0.50	0.5205	1.18	0.9048	1.86	0.9915		
0.52	0.5379	1.20	0.9103	1.88	0.9922		
0.54	0.5549	1.22	0.9155	1.90	0.9928		
0.56	0.5716	1.24	0.9205	1.92	0.9934		
0.58	0.5879	1.26	0.9252	1.94	0.9939		
0.60	0.6039	1.28	0.9297	1.96	0.9944		
0.62	0.6194	1.30	0.9340	1.98	0.9949		
0.64	0.6346	1.32	0.9381				
0.66	0.6494	1.34	0.9419				

7.7 Fracture

i) Fatigue

Manson-Coffin Law

$$\sqrt{N_p} \cdot \epsilon_p = c \quad (c = \text{constant})$$

Miner's Rule

$$\sum \left(\frac{n_i}{N_f} \right) = 1$$

Rayleigh Distribution

$$P(\sigma) = \sigma^{-2} \exp \left\{ -\frac{1}{2} \left(\frac{\sigma}{\sigma_r} \right)^2 \right\}$$

Fraction of peak exceeding

stress (σ) expressed in terms of E

$$E(\sigma) = \exp \left\{ -\frac{1}{2} \left(\frac{\sigma}{\sigma_r} \right)^2 \right\}$$

ii) Fracture Toughness

Stress Intensity $K = Q\sigma\sqrt{\alpha}$

Paris Equation $\frac{da}{dN} = A(\Delta K)^n = A_1 a^{m/2}$ (A, A_1, n and m are constants)

7.8 Some typical values of physical properties

All values are given, unless otherwise stated, for a temperature of 20°C.

	Carbon Steel	Alumi-nium Alloys	Brass 65/35	Copper	Con-crete	Stain-less Steels	Wood
ρ (kg/m ³)	7850	2720	8450	8960	2400	8000	400-800
E (GN/m ²)	207	68.9	105	104	13.8	213	8-13
G (GN/m ²)	79.6	26.5	38.0	46		82	
K (GN/m ²)	172	57.5	115	130		178	
v	0.3	0.3	0.35	0.35	0.1	0.3	
α ($\mu\text{m}/(\text{mK})$)	11	23	19	11.2		18	~ 0.15
σ_y (MN/m ²)	230-460	30-280	62-430	47-320		200-585	
σ_f (MN/m ²)	400-770	90-300	330-530	200-350	27-55	500-800	50-100

K for water is 2.3 GN/m²

The lower values of σ_y and σ_f for carbon and stainless steels refer to materials such as plates and tubes while the higher figures refer to heat-treated material such as used for bolts. The range of values for aluminium, copper and brass is due to the change in material property achieved by heat-treatment and/or mechanical work.

7.6 Selected Values of Error Function $\frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du$

z	$erf z$						
0.00	0.0000	0.68	0.6638	1.36	0.9456	2.00	0.9953
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0.04	0.0451	0.72	0.6914	1.40	0.9523	2.10	0.9970
0.06	0.0676	0.74	0.7047	1.42	0.9554	2.15	0.9976
0.08	0.0901	0.76	0.7175	1.44	0.9583	2.20	0.9981
0.10	0.1125	0.78	0.7300	1.46	0.9611	2.25	0.9983
0.12	0.1348	0.80	0.7421	1.48	0.9637	2.30	0.9989
0.14	0.1570	0.82	0.7538	1.50	0.9661	2.35	0.9991
0.16	0.1790	0.84	0.7651	1.52	0.9684	2.40	0.9993
0.18	0.2009	0.86	0.7761	1.54	0.9706	2.45	0.9995
0.20	0.2227	0.88	0.7867	1.56	0.9726	2.50	0.9996
0.22	0.2443	0.90	0.7969	1.58	0.9746	2.55	0.9997
0.24	0.2657	0.92	0.8068	1.60	0.9764	2.60	0.9998
0.26	0.2869	0.94	0.8163	1.62	0.9780	2.65	0.9998
0.28	0.3079	0.96	0.8254	1.64	0.9796	2.70	0.9999
0.30	0.3286	0.98	0.8342	1.66	0.9811	2.75	0.9999
0.32	0.3491	1.00	0.8427	1.68	0.9825	2.80	0.9999
0.34	0.3694	1.02	0.8508	1.70	0.9838	2.85	0.9999
0.36	0.3893	1.04	0.8587	1.72	0.9850	2.90	1.0000
0.38	0.4090	1.06	0.8661	1.74	0.9861	2.95	1.0000
0.40	0.4284	1.08	0.8733	1.76	0.9872	3.00	1.0000
0.42	0.4475	1.10	0.8802	1.78	0.9882	4.00	1.0000
0.44	0.4662	1.12	0.8868	1.80	0.9891		
0.46	0.4847	1.14	0.8931	1.82	0.9899		
0.48	0.5028	1.16	0.8991	1.84	0.9907		
0.50	0.5205	1.18	0.9048	1.86	0.9915		
0.52	0.5379	1.20	0.9103	1.88	0.9922		
0.54	0.5549	1.22	0.9155	1.90	0.9928		
0.56	0.5716	1.24	0.9205	1.92	0.9934		
0.58	0.5879	1.26	0.9252	1.94	0.9939		
0.60	0.6039	1.28	0.9297	1.96	0.9944		
0.62	0.6194	1.30	0.9340	1.98	0.9949		
0.64	0.6346	1.32	0.9381				
0.66	0.6494	1.34	0.9419				

7.7 Fracture

i) Fatigue

Manson-Coffin Law

$$\sqrt{N_p} \cdot \epsilon_p = c \quad (c = \text{constant})$$

Miner's Rule

$$\sum \left[\frac{n_i}{N_f} \right] = 1$$

Rayleigh Distribution

$$P(\sigma) = \sigma^{-2} \exp \left\{ -\frac{1}{2} \left(\frac{\sigma}{\sigma_r} \right)^2 \right\}$$

Fraction of peak exceeding

stress (σ) expressed in terms of E

$$E(\sigma) = \exp \left\{ -\frac{1}{2} \left(\frac{\sigma}{\sigma_r} \right)^2 \right\}$$

ii) Fracture Toughness

Stress Intensity $K = Q\sigma\sqrt{\pi a}$

Paris Equation $\frac{da}{dN} = A(\Delta K)^n = A_1 a^{m/2} \quad (A, A_1, n \text{ and } m \text{ are constants})$

7.8 Some typical values of physical properties

All values are given, unless otherwise stated, for a temperature of 20°C.

	Carbon Steel	Alumi-nium Alloys	Brass 65/35	Copper	Concrete	Stain-less Steels	Wood
ρ (kg/m ³)	7850	2720	8450	8960	2400	8000	400-800
E (GN/m ²)	207	68.9	105	104	13.8	213	8-13
G (GN/m ²)	79.6	26.5	38.0	46		82	
K (GN/m ²)	172	57.5	115	130		178	
v	0.3	0.3	0.35	0.35	0.1	0.3	
α ($\mu\text{m}/(\text{mK})$)	11	23	19	11.2		18	~ 0.15
σ_y (MN/m ²)	230-460	30-280	62-430	47-320		200-585	
σ_f (MN/m ²)	400-770	90-300	330-530	200-350	27-55	500-800	50-100

K for water is 2.3 GN/m²

The lower values of σ_y and σ_f for carbon and stainless steels refer to materials such as plates and tubes while the higher figures refer to heat-treated material such as used for bolts. The range of values for aluminium, copper and brass is due to the change in material property achieved by heat-treatment and/or mechanical work.

Metals

Property	Copper	Iron
Crystal structure	f.c.c.	b.c.c.
Bonding	metallic	metallic
Lattice constant (\AA)	3.61	2.86
Atomic volume ($\text{m}^3/\text{kg mol}$)	7.09×10^{-3}	7.10×10^{-3}
ρ (kg/m^3)	8.96×10^3	7.87×10^3
Resistivity ($\Omega \text{ m}$)	1.72×10^{-8}	10×10^{-8}
Cohesive energy ($\text{J}/\text{kg mol}$)	3.38×10^8	4.05×10^8
Melting point ($^{\circ}\text{C}$)	1083	1530
$\alpha(\mu\text{m}/(\text{mK}))$	16.7	12.1
Fermi energy (eV)	7.04	11.2
Work function (eV)	4.07 - 4.18	3.91 - 4.77
Temperature coefficient of resistance (K^{-1})	+0.0043	+0.0065
Effective radius (\AA) of		
(a) neutral atom	1.27	1.26
(b) singly charged ion	0.96	-
(c) doubly charged ion	0.70	0.75

Semiconductors

Property	Germanium	Silicon
Crystal structure	diamond	diamond
Bonding	covalent	covalent
Lattice constant (\AA)	5.6575	5.4307
Atomic volume ($\text{m}^3/\text{kg mol}$)	13.5×10^{-3}	12.0×10^{-3}
Density (kg/m^3)	5.32×10^3	2.33×10^3
Cohesive energy (J/kg mol)	3.72×10^8	4.39×10^8
Melting point ($^\circ\text{C}$)	958.5	1412
Mobility ($\text{m}^2/(\text{V s})$)	{ electrons 0.38 holes 0.18	{ electrons 0.19 holes 0.05
Energy gap (eV) (room temperature)	0.67	1.107
Density of states effective mass	{ electrons 0.35 me holes 0.56 me	{ electrons 0.58 me holes 1.06 me
$\alpha(\mu\text{m}/(\text{mK}))$	5.75	7.6

polymers

<u>PROPERTY</u>	Polyethylene (H.D)	Polyvinyl Chloride	Polystyrene
Polymer Structure	$\text{[---CH}_2\text{---CH}_2\text{---]}_n$	$\left[\text{---CH}_2\text{---CH}(\text{Cl})\text{---}\right]_n$	$\left[\text{---CH}_2\text{---CH}(\text{C}_6\text{H}_5)\text{---}\right]_n$
Structural State	Crystalline	Amorphous/ Slightly Crystalline	Amorphous/ Crystalline
$\rho(\text{kg/m}^3)$	0.96×10^3	1.7×10^3	1.05×10^3
Resistivity (Ωm)	$10^6 - 10^{10}$	10^5	10^{10}
$\alpha(\mu\text{m/mK})$	120	190	63
$E(\text{GN/m}^2)$	70-280	2500-3500	3500-4200
$\sigma_f(\text{MN/m}^2)$	7-14	28-40	35-50
Tg(K)	153	353	373
<u>PROPERTY</u>	Poly(methyl-methacrylate)	Polytetrafluoroethylene	Polyisoprene (Natural Rubber)
Polymer Structure	$\left[\text{---CH}_2\text{---}\overset{\text{CH}_3}{\underset{\text{O---C---O---CH}_3}{\text{C}}\text{---}\right]_n$	$\left[\text{---CF}_2\text{---CF}_2\text{---}\right]_n$	$\left[\text{---CH}_2\text{---}\overset{\text{CH}_3}{\underset{\text{CH---CH---CH}_2}{\text{C}}\text{---}\right]_n$
Structural State	Amorphous	Crystalline	Elastomer
$\rho(\text{kg/m}^3)$	1.2×10^3	2.2×10^3	1.5×10^3
Resistivity (Ωm)	10^8	10^8	$10^5 - 10^7$
$\alpha(\mu\text{m/mK})$	90	100	-
$E(\text{GN/m}^2)$	2500-4000	400-650	7-70
$\sigma_f(\text{MN/m}^2)$	50-70	14-30	2-10
Tg(K)	380	399	203
<u>PROPERTY</u>	Nylon 6:6	Phenol-Formaldehyde Resin (Bakelite)	
Polymer Structure	$\left[\text{---NH---(CH}_2\text{)}_6\text{---NH---C}(=\text{O})\text{---(CH}_2\text{)}_4\text{---C}(=\text{O})\text{---}\right]_n$	$\left[\text{---CH}_2\text{---}\overset{\text{OH}}{\underset{\text{CH}_2}{\text{C}}\text{---CH}_2\text{---}\overset{\text{OH}}{\underset{\text{CH}_2}{\text{C}}\text{---CH}_2\text{---}\right]_n$	
Structure State	Crystalline	Amorphous	
$\rho(\text{kg/m}^3)$	1.15×10^3	1.3×10^3	
Resistivity (Ωm)	10^6	10^4	
$\alpha(\mu\text{m/mK})$	100	72	
$E(\text{GN/m}^2)$	2000-3000	7000	
$\sigma_f(\text{MN/m}^2)$	50-70	50	
Tg(K)	323	-	

Periodic Table of the Elements

1. The atomic number indicates the number of protons in the nucleus of an atom. In the neutral atom these protons are electrically balanced by an equal number of electrons outside the nucleus. Only neutral atoms are considered in the Periodic Classification.

2. Electrons travel far from the nucleus but if those regions where they spend most of their time are considered, a well-defined pattern of layers or 'Principal Shells' appears. Each shell is known by a Principal Quantum Number 1, 2, 3...7 or sometimes by the letters K, L, M...etc.

3. In each shell the electrons move around the nucleus in complicated, three-dimensional patterns called Orbitals. The laws of Quantum Mechanics permit only certain types of orbital. An electron following one of these paths possesses an amount of energy (Energy Level) characteristic of that orbital.

4. Four types of orbital are encountered; they are identified by the letters s, p d and f. s is the simplest whilst p, d and f are progressively more complex.

5. The number of orbitals per shell increases with shell number. (See the lower diagram overleaf.) The first contains only an s orbital, the second an s and three p's, the third adds five d orbitals and the fourth seven f's. These groups of like orbitals in any Principal Shell are called s p d or f sub-shells.

Each sub-shell, depending on its principal quantum number and type, has a characteristic energy the order of which is generally proportional to the distance of the sub-shell from the nucleus.

6. Each orbital accepts either one or two electrons and the maximum number of electrons per sub-shell is shown on the diagram.

7. Electrons take positions in orbitals where the energy level is lowest. Up to element 18 (Argon) sub-shells and shells are built in an orderly sequence to maximum capacity. But in the next group the order changes because it happens that the energy level of the 4 s state is a little lower than that of the 3 d state.

8. The first transition series begins with Scandium (element 21) where the energy levels of the 4 s and 3 d orbitals are so nearly equal that there is a tendency for electrons to move from one orbital to another, causing variable valency. The same happens in the fifth period with 5 s and 4 d orbitals and in the sixth period with the 6 s and 5 d orbitals.

9. In the Lanthanide and Actinide series of elements, the 4 f and 5 f orbitals are occupied only after the s p d and s orbitals outside them have filled or begun to fill. The effect upon the chemistry of the elements is very small because the f orbitals are deep in the core of the atom. For this reason there is little difference between one element and its immediate neighbours.

10. In any element, the so called Valency Electrons are those moving in orbitals of the highest energy levels. In this Chart of the Periodic Classification, the number and position of the valency electrons is indicated in the boxes underneath the various columns e.g. Rhodium—element 45—has nine valency electrons; 8 in the 4 d sub-shell and 1 in the 5 s.

The particular sub-shell being filled with electrons is shown by the figures 4, 3, 2, 1, 4, 5, 6, 7 in front of the rows of elements e.g. the 3 d in front of elements 21—30.

NOTES

1. The following atomic weights are based on the exact number 12 for the carbon isotope 12, as agreed between the International Unions of Pure and Applied Physics and of Pure and Applied Chemistry, 1961.

2. The values given normally indicate the mean atomic weight of the mixture of isotopes found in nature. Particular attention is drawn to the value for hydrogen, boron, carbon, oxygen, silicon and sulphur, where the deviation shown is due to variation in relative concentration of isotopes.

Symbol	Name	Atomic Number	Atomic Weight	Symbol	Name	Atomic Number	Atomic Weight
A or Ar	Argon	18	39.948	Mg	Magnesium	12	24.312
Ac	Actinium	89	—	Mn	Manganese	25	54.9380
Ag	Silver	47	107.870	Mo	Molybdenum	42	95.94
Al	Aluminium	13	26.9815	N	Nitrogen	7	14.0067
Am	Americium	95	—	Na	Sodium	11	22.9898
As	Arsenic	33	74.9216	Nb	Niobium	41	92.906
At	Astatine	85	—	Nd	Neodymium	60	144.24
Au	Gold	79	196.967	Ne	Neon	10	20.183
B	Boron	5	10.811	Ni	Nickel	28	58.71
		±0.003		No	Nobelium	102	—
Ba	Barium	56	137.34	Np	Neptunium	93	—
Be	Beryllium	4	9.0122	O	Oxygen	8	15.9994
Bi	Bismuth	83	208.980	Os	Osmium	76	190.2
Bk	Berkelium	97	—	P	Phosphorus	15	30.9738
Br	Bromine	35	79.909	Pa	Protactinium	91	—
C	Carbon	6	12.01115	Pb	Lead	82	207.19
		±0.00005		Pd	Palladium	46	106.4
Ca	Calcium	20	40.08	Pr	Praseodymium	59	140.907
Cd	Cadmium	48	112.40	Pt	Platinum	78	195.09
Ce	Cerium	58	140.12	Pu	Plutonium	94	—
Cf	Californium	98	—	Ra	Radium	88	—
Cl	Chlorine	17	35.453	Rb	Rubidium	37	85.47
Gm	Curium	96	—	Re	Rhenium	75	186.2
Co	Cobalt	27	58.9332	Rh	Rhodium	45	102.905
Cr	Chromium	24	51.996	Rn	Radon	86	—
Cs	Caesium	55	132.905	Ru	Ruthenium	44	101.07
Cu	Copper	29	63.54	S	Sulphur	16	32.064
Dy	Dysprosium	66	162.50	Sb	Antimony	51	121.75
Er	Erbium	68	167.26	Sc	Scandium	21	44.956
Es	Einsteinium	99	—	Se	Selenium	34	78.96
Eu	Europium	63	151.96	Si	Silicon	14	28.086
F	Fluorine	9	18.9984	Sm	Samarium	62	150.35
Fe	Iron	26	55.847	Tn	Tin	50	118.69
Fm	Fermium	100	—	Sr	Strontium	38	87.62
Fr	Francium	87	—	Ta	Tantalum	73	180.948
Ga	Gallium	31	69.72	Tb	Terbium	65	158.924
Gd	Gadolinium	64	157.25	Tc	Technetium	43	—
Ge	Germanium	32	72.59	Te	Tellurium	52	127.60
H	Hydrogen	1	1.0079	Th	Thorium	90	232.038
		±0.00001	Ti	Titanium	22	47.90	
He	Helium	2	4.0026	Tl	Thallium	81	204.37
Hf	Hafnium	72	178.49	Tm	Thulium	69	168.934
Hg	Mercury	80	205.59	U	Uranium	92	238.03
Ho	Holmium	67	164.930	V	Vanadium	23	50.942
I	Iodine	53	126.9044	W	Tungsten	74	183.85
In	Indium	49	114.82	Xe	Xenon	54	131.30
Ir	Iridium	77	192.2	Y	Yttrium	39	88.905
K	Potassium	19	39.102	Yb	Ytterbium	70	173.04
Kr	Krypton	36	83.80	Zn	Zinc	30	65.37
La	Lanthanum	57	138.91	Zr	Zirconium	40	91.22
Li	Lithium	3	6.939				
Lu	Lutetium	71	174.97				
Md	Mendelevium	101	—				

B. THERMODYNAMICS AND FLUID MECHANICS

B.1 Thermodynamic Relationships

1st Law

$$dQ - dW = dU$$

Enthalpy

$$H = U + pV \quad \text{or} \quad h = u + pv$$

For reversible process

$$dS = \left[\frac{dQ}{T} \right]_{\text{rev}}$$

" "

$$dW = pdV$$

Helmholtz function

$$F = U - TS \quad \text{or} \quad f = u - Ts$$

Gibbs function }

$$G = H - TS \quad \text{or} \quad g = h - Ts$$

From 1st Law for a homogeneous fluid

$$Tds = du + pdv = dh - vdp$$

Specific heat at constant volume

$$c_v = \left[\frac{\partial u}{\partial T} \right]_v$$

Specific heat at constant pressure

$$c_p = \left[\frac{\partial h}{\partial T} \right]_p$$

Specific heat ratio

$$\gamma = c_p/c_v$$

Reversible engine (Carnot) efficiency

$$\eta = 1 - (T_{\text{sink}}/T_{\text{source}})$$

Engine indicated Power

$$P_i = p_m V_s N_c$$

Steady flow energy equation

$$(Q-W)/m = h_2 - h_1 + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)$$

Continuity equation

$$\dot{m} = \rho A C$$

General relationships for a perfect gas:

$$pv = RT$$

$$pV_o = R_o T$$

$$MR = R_o$$

$$\Delta U = mc_v(T_2 - T_1)$$

$$\Delta H = mc_p(T_2 - T_1)$$

$$\Delta S = mc_v \ln \left(\frac{T_2}{T_1} \right) + mc_p \ln \left(\frac{P_2}{P_1} \right)$$

$$c_p - c_v = R \quad \frac{\gamma-1}{\gamma} = \frac{R}{c_p}$$

Van der Waals' equation $(p + \frac{a}{v^2})(v - b) = RT$

$$S = k \ln P \quad k = R_o/N$$

Availability, (closed system): $(A_1 - A_0) = (U_1 + p_0 V_1 - T_0 S_1) - (U_0 + p_0 V_0 - T_0 S_0)$

" (flow process) : $(B_1 - B_0) = (H_1 - T_0 S_1) - (H_0 - T_0 S_0)$

Maximum work of a Reaction $W_{\max} = G_{\text{react}} - G_{\text{prod}} = R_o T \ln(K_p \cdot P^n)$

For reversible polytropic ($pV^n = \text{constant}$) closed system:
 $W = (p_1 V_1 - p_2 V_2)/(n-1)$

For perfect gas also:

$$W = mR(T_1 - T_2)/(n-1) \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{\frac{n-1}{n}}$$

$$Q = \frac{V_1 - V_2}{T_1 - T_2} \cdot n \cdot R$$

For adiabatic reversible (isentropic reversible):
 $n = \gamma$

For isothermal reversible:

$$W = Q = pV \ln\left(\frac{V_1}{V_2}\right) \quad (n = 1)$$

Maxwell relations

$$\begin{aligned} \left(\frac{\partial T}{\partial P}\right)_S &= -\left(\frac{\partial P}{\partial S}\right)_V & \left(\frac{\partial T}{\partial P}\right)_S &= \left(\frac{\partial V}{\partial S}\right)_P \\ \left(\frac{\partial P}{\partial T}\right)_V &= \left(\frac{\partial S}{\partial V}\right)_T & \left(\frac{\partial V}{\partial T}\right)_P &= -\left(\frac{\partial S}{\partial P}\right)_T \end{aligned}$$

Heat transfer

Conduction (one dimensional)
 $\dot{Q}/A = -k dT/dx$
 $= k(T_1 - T_2)/x_{1,2}$

" (radial flow)
 $\dot{Q}/A = 2\pi k \Delta T / \ln(r_2/r_1)$

Forced convection in a tube
 $Nu = 0.023 Re^{0.8} Pr^{0.4}$
 (characteristic length = hydraulic mean diameter) (see 8.5)

Log. mean temperature difference
 $\frac{\Delta T_{in} - \Delta T_{out}}{\ln(\Delta T_{in}/\Delta T_{out})} = \Delta T_m$

Stefan-Boltzmann Law of radiation
 $q_b = \sigma T^4$

Radiation exchange:

Grey body to black or large enclosure
 $\dot{Q}/A = \epsilon_1 [T_1^4 - T_2^4]$

Large parallel grey surfaces
 $\dot{Q}/A = \frac{\sigma [T_1^4 - T_2^4]}{T_1/\epsilon_1 + 1/\epsilon_2 - 1}$

Heat transfer coefficient $h = \dot{Q}/A\Delta T$
 emissivity $\epsilon = q/q_b$

Fluid Mechanics

Statics

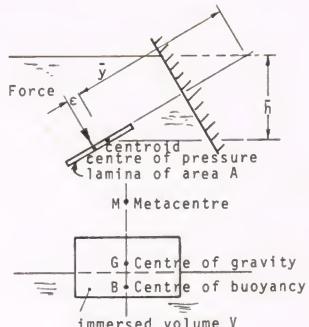
$$\frac{\partial p}{\partial z} = -\rho g$$

$$\text{Force} = \rho g A h$$

$$\epsilon = \frac{(Ak^2)}{Ay} \text{ centroid}$$

$$\overline{GM}_{roll} = \frac{Ak^2}{V} - \overline{BG}$$

(Ak^2 is 2nd moment of area about rolling axis)



Dynamics

For simple Newtonian flow
 $\tau = u \frac{dy}{dx}$

Euler's equation
 $\frac{1}{\rho} \frac{dp}{dx} + c \frac{dc}{dx} + g \frac{dz}{dx} = 0$

Bernoulli's equation
 $\frac{p}{\rho g} + \frac{c^2}{2g} + z = \text{constant}$

For constant area flow with friction (Fanno)

$$\frac{dp}{\rho} + c \frac{dc}{dx} + \frac{2fc^2}{D} dx = 0$$

Acceleration along a stream-line
 $a_s = v_s \frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t}$

Acceleration normal to a streamline
 $a_n = \frac{v_s^2}{r} + \frac{\partial v_n}{\partial t}$

Reynolds' Equation for bearings:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12n} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12n} \frac{\partial p}{\partial y} \right) = \frac{1}{2} \frac{\partial}{\partial x} (\rho Uh) + \frac{\partial}{\partial t} (\rho h) + \rho w$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\rho h^3}{12n} \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(\frac{\rho h^3}{12n} \frac{\partial p}{\partial \phi} \right) = \frac{1}{2} \frac{\partial}{\partial \phi} (\rho h \omega) + \frac{\partial}{\partial t} (\rho h) + \rho w$$

Hydraulic machines

Head coefficient
 $\psi = Y/\omega^2 D^2$

Flow
 $\phi = Q/\omega D^3$

Dimensionless specific speed
 $n_s = \left| \omega Q^{\frac{1}{2}} / Y^{\frac{3}{2}} = \phi^{\frac{1}{2}} / \psi^{\frac{3}{2}} = \frac{\omega (\text{Power})^{\frac{1}{2}}}{\rho^{\frac{1}{2}} Y^{\frac{5}{2}}} \right|_{\text{MAX}}$

Dimensionless diameter	$\Delta = DY^{\frac{1}{2}}/Q^{\frac{1}{2}}$
Dimensionless suction specific speed	$N_{SS} = \omega Q^{\frac{1}{2}} / (NPSE)^{\frac{3}{2}}$
Cavitation number	σ or $k = (P_{\infty} - P_v) / (\frac{1}{2}\rho V_{\infty}^2)$, suffix ∞ , reference condition.
Cavitation number (Thoma)	$\sigma_{Th} = (P_1 - P_v) / (P_2 - P_1)$ suffix 1, abs. pressure at 1p side of machine; suffix 2, abs. pressure at hp side of machine; suffix v, vapour pressure

Open Channel Hydraulics

Chézy equation $V = C\sqrt{RS}$

Manning equation: $V = \frac{1}{n} R^{2/3} S^{1/2}$

Steady gradually varied flow equation: $\frac{dA}{dx} = \frac{S_o - S_f}{1 - \alpha \frac{V^2}{2g}}$ (rectangular channel)

Unsteady gradually varied flow equation: $\frac{\partial A}{\partial x} + \frac{V}{g} \frac{\partial V}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} = S_o - S_f$ no local inflow or outflow

Continuity equation: $A \frac{\partial V}{\partial x} + V \frac{\partial A}{\partial x} + T \frac{\partial d}{\partial t} = 0$ outflow

Conjugate depths in hydraulic jump: $\frac{d_2}{d_1} = \frac{1}{2} \left[\sqrt{1+8F_1^2} - 1 \right]$ (rectangular channel)

High speed gas flow

Nozzles:

Mass flow given by $\dot{m} = AC_d \sqrt{\frac{2n}{(n-1)}} p_0 p_0 \left[\left(\frac{p_1}{p_0}\right)^2/n - \left(\frac{p_0}{p_1}\right)^{n+1} \right]$

Critical pressure ratio $\frac{p_1^*}{p_0} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Sonic velocity $a = \sqrt{np/p}$

where $n \approx 1.3$ for steam, initially superheated
 ≈ 1.135 for steam, initially wet or dry saturated
 $= \gamma \approx 1.4$ for air

For perfect gas:

Stagnation temperature $T_0 = T \left[1 + \frac{(Y-1)}{2} M^2 \right]$

For air in isentropic flow ($\gamma = 1.4$) $\frac{\dot{m} T_0}{A^* p_0} = 0.0404 \frac{\text{kg}}{\text{Ns}}$

Turbine Ellipse Law $\frac{\dot{m} \sqrt{T_0}}{A p_0} = \left[1 - \left(\frac{p}{p_0} \right)^2 \right]^{\frac{1}{2}}$

8.5 Dimensionless groups

Drag coefficient	$C_D = \text{drag force}/\frac{1}{2} \rho V^2 A$
Discharge coefficient	$C_d = Q_{\text{actual}} / \left\{ A_{\text{throat}} \left[\frac{2 \Delta p_{\text{meter}} / \rho}{1 - \left(\frac{A_{\text{throat}}}{A_{\text{pipe}}} \right)^2} \right] \right\}^{\frac{1}{2}}$
Fourier number	$Fo = (k/\rho c_p) t/L^2$
Froude number	$Fr = V/\sqrt{gL}$
Grashof number	$Gr = g \beta \Delta T L^3 \rho^2 / \mu^2$
Mach number	$M = V/a$
Nusselt number	$Nu = hL/k$
Prandtl number	$Pr = \mu c_p / k$
Reynolds' number - general	$Re = \rho VL / \mu$
" " - rotating disc	$Re = \rho \omega D^2 / 4\mu$
Weber number	$We = V(\rho \ell / \sigma)^{\frac{1}{2}}$
Pipeflow friction factor	$f = gDh_f / 2 \ell V^2$ (round pipes) $2 g m_h f / \ell V^2$ (non-circular duct)
Wall shear stress coefficient f_w	$f_w = \tau_w / \frac{1}{2} \rho V^2$

8.6 Composition of air

	Vol. Analysis	Grav. Analysis
Nitrogen (N_2 - 28.013)	0.7809	0.7553
Oxygen (O_2 - 31.999)	0.2095	0.2314
Argon (A_r - 39.948)	0.0093	0.0128
Carbon dioxide (CO_2 - 44.010)	0.0003	0.0005

Mean Molecular Weight $M = 28.96$

Specific Gas Constant $R = 0.2871 \text{ kJ/(kgK)}$

8.7 Temperatures at the primary fixed points

Normal boiling point of oxygen (oxygen point)	-182.97°C
Triple point of water	0.01°C
Normal boiling point of water (steam point)	100.00°C
Normal boiling point of sulphur (sulphur point)	446.6°C
Normal melting point of silver (silver point)	960.8°C
Normal melting point of gold (gold point)	1063°C

(iii) Solids

8.8 Critical constants

	molecular weight	T _c (K)	P _c (bar) (10 ⁵ N/m ²)	P _c (kg/m ³)
hydrogen	2.02	33.3	13.0	31
helium (4)	4.00	5.3	2.29	69.3
water vapour	18.02	647.30	221.2	318.3
nitrogen	28.01	126.1	33.9	311
oxygen	32.00	154.4	50.4	430
carbon dioxide	44.01	304.15	73.8	468

8.9 Approximate physical properties at 20°C, 1 bar (10⁵N/m²)

	R kJ/kgK	ρ kg/m ³	c _p kJ/kgK	c _p /c _v	μ mNs/m ² = cP	k W/mK
hydrogen	4.16	0.082	14.3	1.40	8.8x10 ⁻³	1.8x10 ⁻¹
helium	2.08	0.164	5.23	1.66	1.96x10 ⁻²	1.4x10 ⁻¹
nitrogen	0.294	1.16	1.04	1.40	1.76x10 ⁻²	2.6x10 ⁻²
oxygen	0.260	1.31	0.91	1.40	2.03x10 ⁻²	2.6x10 ⁻²
carbon dioxide	0.190	1.80	0.84	1.28	1.47x10 ⁻²	1.7x10 ⁻²
air	0.287	1.19	1.005	1.40	1.82x10 ⁻²	2.6x10 ⁻²

(ii) Liquids

	ρ kg/m ³	c _p kJ/(kgK)	μ cP	k W/(mK)	σ N/m	β 10 ⁻³ K ⁻¹
water	1,000	4.19	1.002	0.6	0.073	0.21
mercury	13,600	0.14	1.55	8.7	0.51	0.18
castor oil	960	2.20	1000	0.18	0.039	
benzene	880	1.80	0.656	0.16	0.029	
ethyl alcohol	790	2.86	1.20	0.19	0.022	1.08
engine oil~	890	1.9	80	0.15	-	0.8
Freon 12	1,350	0.96	0.273	0.073	-	

	ρ kg/m ³	c _p kJ/(kgK)	k W/(mK)	α μm/(mK)
duralumin	2720	0.88	170	23
mild steel	7850	0.46	52	11
stainless steel (18% Ni, 8% Cr)	7810	0.46	16	18
brass (65/35)	8450	0.37	120	19
concrete	2400	0.88	1.1	10-14
wood (pine)	500	2.8	0.15	0.15
firebrick	170	0.81	0.38	3-9

(iv) Fuels

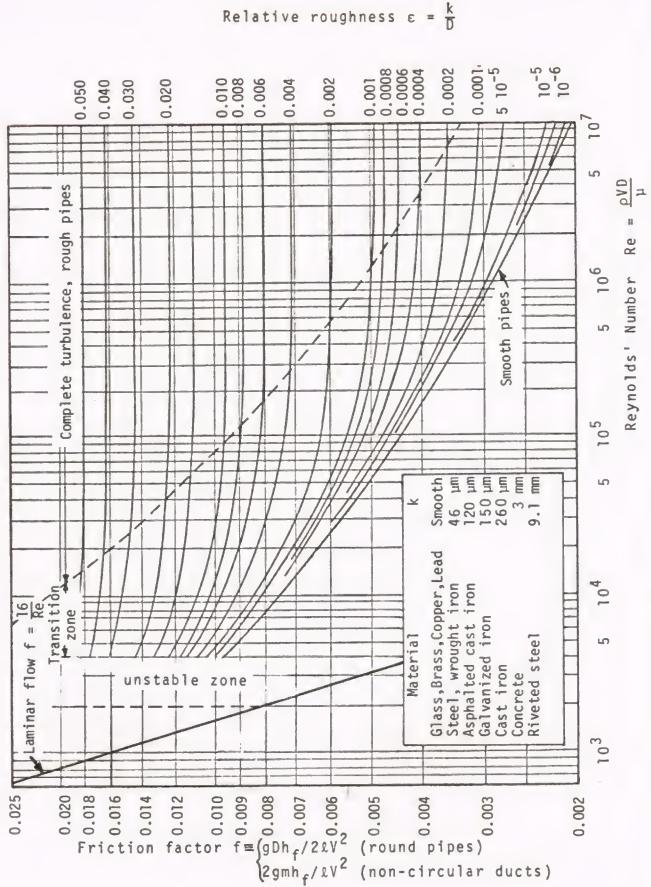
(a) Gases (fuels)

	Composition by Volume							Relative Density (Air) = 1	Calorific Value MJ/m ³ 15°C 1.01325 bar	Theoretical Air	
	N ₂	H ₂	CH ₄	C ₂ H ₆	C ₃ H ₈	C ₄ H ₁₀	C ₃ H ₆				
	Gross	Net	Vol/Vol								
Hydrogen	100							0.0696	12.10	10.22	2.38
Methane		100						0.5537	37.71	33.95	9.52
North Sea Gas	1.5	94.4	3.0	0.5	0.2			0.589	38.62	34.82	9.75
Propane*			1.5	91.0	2.5	5.0	1.523	93.87	86.43	23.76	
Butane*		0.1	0.5	7.2	87.0	4.2	1.941	117.75	108.69	29.92	

* Commercial Liquid petroleum Gas (L.P.G.) See also data on liquid fuels below.

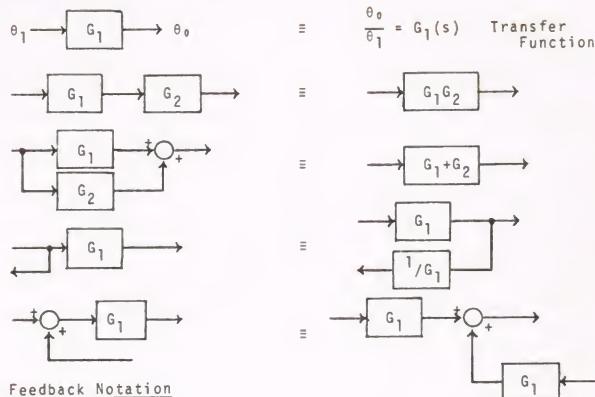
(b) Liquids (fuels), typical values

	Composition % Mass			Density at 15°C kg/m ³	Calorific Value MJ/kg at 15°C	
	C	H	S		Gross	Net
Propane*	82.0	18.0			505	50.0
Butane*	81.9	17.0			575	49.3
Petrol	85.5	14.4	0.1	733	46.9	43.7
Kerosene	85.9	14.0	0.1	780	46.5	43.4
Diesel (Gas Oil)	85.7	13.4	0.9	840	45.4	42.4

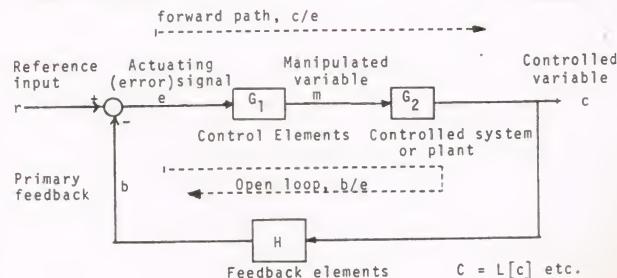


9. AUTOMATIC CONTROL

1. Block Diagrams



2. Feedback Notation



$$C = L[c] \text{ etc.}$$

$$\text{Input/Output closed loop transfer function} = \frac{C}{R} = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

$$\text{Input/Error closed loop transfer function} = \frac{E}{R} = \frac{1}{1 + G_1 G_2 H}$$

$$= 1 + G_1 G_2 H = 0$$

Characteristic equation

$$G_1 G_2 \triangleq KGH = \frac{K}{s^n} \prod_{i=1}^m (s + z_i)$$

$$= \frac{K}{s^n} \prod_{i=1}^{n-1} (s + p_i)$$

$-z_i, p_i$ zeros and poles
 n system type number
 n system order number

3. Stability Criteria for Linear Systems

3.1 Root location: No closed loop system pole may have positive real part

3.2 Routh Array

Characteristic equation $a_n s^n + a_{n-1} s^{n-1} \dots + a_1 s + a_0 = 0$

	1	2	3	"
n	a_n	a_{n-2}	a_{n-4}	"
n-1	a_{n-1}	a_{n-3}	a_{n-5}	"
n-2	b_1	b_2	b_3	"
n-3	c_1	c_2	c_3	"
"	"	"	"	"
$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$	$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$			
$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1}$	$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$	etc.		

Number of closed loop poles with positive real part = number of sign changes in column 1.

3.3 Nyquist Encirclement

$$P = N + Z$$

N = number of clockwise encirclements of $(-1, j0)$ by open loop locus

P = number of closed loop poles with positive real part

Z = number of open loop poles with positive real part

3.4 Gain Margin = $|KG(j\omega_g)H(j\omega_g)|^{-1}$, ω_g such that

$$\angle KG(j\omega_g)H(j\omega_g) = -180^\circ$$

3.5 Phase Margin = $180^\circ + \angle KG(j\omega_p)H(j\omega_p)$, ω_p such that

$$|KG(j\omega_p)H(j\omega_p)| = 1$$

4. Rules of Root Locus Sketching

4.1 Every point, α , on the root locus for positive K satisfies

$$|G(\alpha)H(\alpha)| = 1/K$$

$$\angle G(\alpha)H(\alpha) = (1+2k)180^\circ \quad k = 0, \pm 1, \pm 2, \dots$$

4.2 The number of branches of the root locus is equal to the number of poles.

4.3 Branches of the locus can be considered to start on the poles ($K = 0$) and terminate on zeros ($K = \infty$).

4.4 Points of the root locus exist on the real axis to the left of an odd number of poles plus zeros.

4.5 The locus is symmetrical with respect to the real axis.

4.6 The angles of asymptotes, α_k , to the root locus are given by

$$\alpha_k = \frac{\pm(2k+1)\pi}{n-m} \quad k = 0, 1, 2, \dots$$

4.7 The intersection of the asymptotes and the real axis occurs at s_r

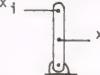
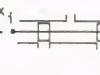
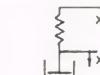
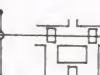
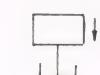
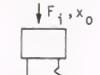
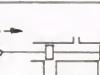
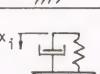
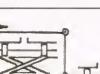
$$s_r = -\frac{\sum p_i - \sum z_i}{n-m}$$

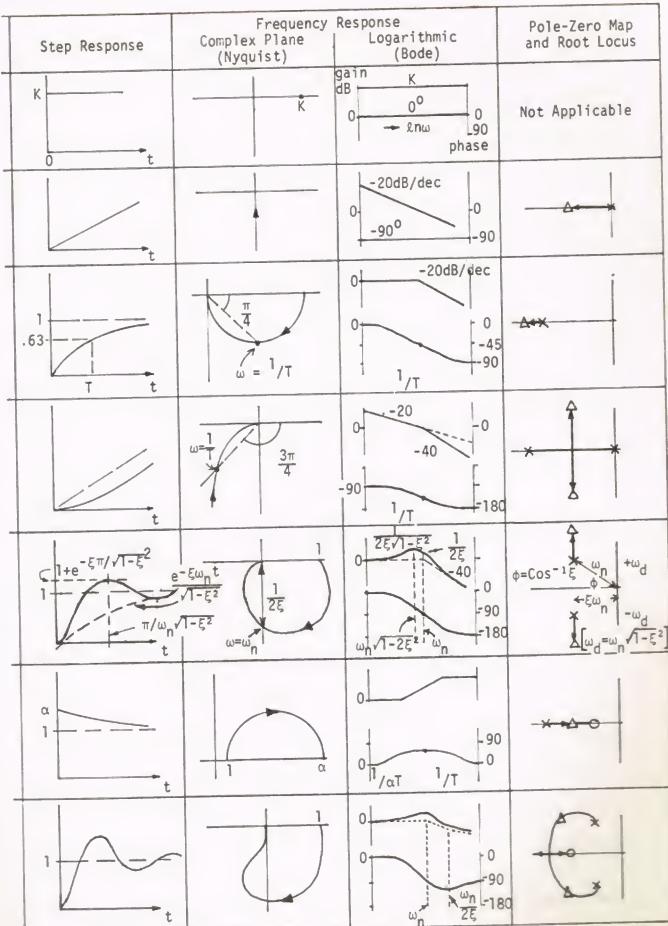
4.8 The locus leaves the real axis or arrives at it at points α where α is given by

$$\frac{d}{d\alpha} [\ln\{KG(\alpha)H(\alpha)\}] = 0$$

4.9 The intersection of the root locus and the imaginary axis can be found by application of Routh's Stability criteria.

5. TABLE OF CHARACTERISTIC SYSTEMS

Typical Example			Basic form of transfer function $G(s)$
Electrical	Dynamic	Hydraulic	
			K
			$\frac{1}{s}$
			$\frac{1}{T+s}$
			$\frac{1}{s(1+Ts)}$
			$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$
			$\frac{1+aTs}{1+Ts}$
			$\frac{\omega_n^2 (1+2\xi s/\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2}$



10. ELECTRICITY

Ohm's Law

$$V = IR, \quad R = \frac{V}{I}, \quad I = \frac{V}{R}$$

Power

$$\text{DC Power} = VI = I^2R = V^2/R$$

$$\text{AC Power} = \text{Re}(V \cdot I) = |V||I|\cos\phi$$

Resistance

$$I = \frac{a}{\rho_0(1+\alpha T)} \frac{dV}{dx} \quad R = \int \frac{\rho_0(1+\alpha T)}{a} dx$$

Inductance

$$e = -L \frac{di}{dt} \quad i = - \int \frac{V}{L} dt$$

$$L = N^2 \mu_0 \mu_r a / l$$

for L R circuit decay $i = I e^{-Rt/L}$

$$\text{Stored energy} = \frac{1}{2} L I^2$$

Capacitance

$$Q = CV = \int idt$$

$$i = \frac{dq}{dt} = C \frac{dv}{dt}$$

$$C = \epsilon_0 \epsilon_r (n-1) a/d, \text{ for } n \text{ parallel plates}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F m}^{-1}$$

for RC circuit discharge $i = -I e^{-t/RC}$

$$\text{Stored energy} = \frac{1}{2} C V^2$$

$$F = \frac{1}{2} \epsilon_0 \epsilon_r a \left[\frac{V}{x} \right]^2$$

Electrostatics

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$F = eE = -e \text{ grad}V$$

$$Q = \int D \cdot dS (= \psi)$$

$$D = \epsilon_0 \epsilon_r E$$

Electromagnetism

$$E = -N \frac{d\phi}{dt}$$

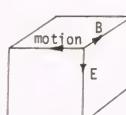
$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = BIL$$

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$\frac{dH}{dx} = \frac{IS \sin \alpha}{4\pi x^2}$$

$$\text{For solenoid } H = \frac{NI}{l}$$



Magnetism

$$H = \frac{B}{\mu_0 \mu_r}$$

For a magnetic circuit

$$B = \frac{\Phi}{a}$$

$$\Phi = \frac{NI}{\mu_1 a_1 + \mu_2 a_2}$$

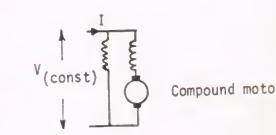
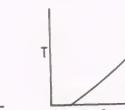
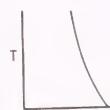
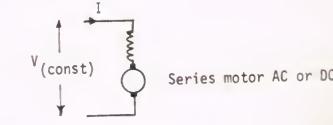
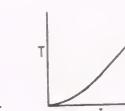
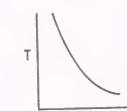
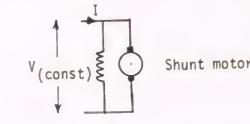
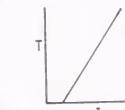
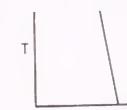
$$\text{Stored Energy Density} = \frac{1}{2} HB = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$F = (\frac{1}{2} HB) a = \frac{B^2 a}{2 \mu_0}$$

DC Machines

$$E = \frac{2Z}{c} \frac{n}{60} p \phi \quad T = \frac{I_a Z p \phi}{2\pi c}$$

$$V = E \pm I_a R_a$$

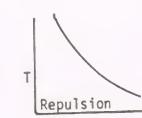
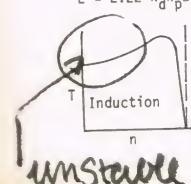


AC Machines

$$\text{Synchronous speed} = f/p$$

$$E = 2.22 k_d p Z f \phi_{\text{rms}}$$

$$T \propto \frac{\phi^2 s R}{R^2 + (sX_0)^2}$$



AC Circuits $V_{rms} = \frac{1}{\sqrt{2}} V_{max}$

Series LCR

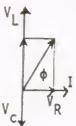
$$Z = (R^2 + (\omega L - \frac{1}{\omega C})^2)^{\frac{1}{2}}$$

$$\omega = 2\pi f$$

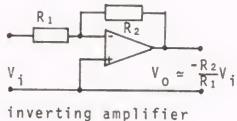
$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$\cos\phi = \frac{R}{Z}$$

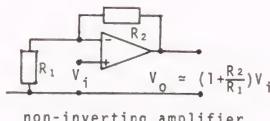
$$\text{At resonance } \omega = \omega_0 = \frac{1}{\sqrt{LC}} \quad Q \text{ factor} = \omega_0 \frac{L}{R}$$



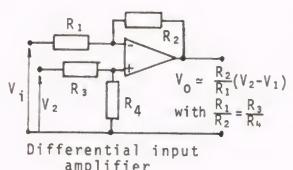
Basic Op'Amp' Circuits



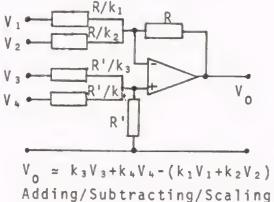
inverting amplifier



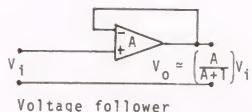
non-inverting amplifier



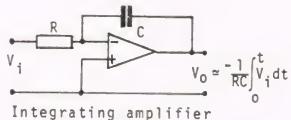
Differential input amplifier



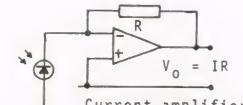
Adding/Subtracting/Scaling



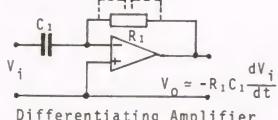
Voltage follower



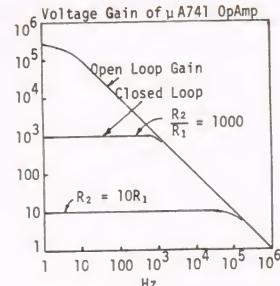
Integrating amplifier



Current amplifier



Differentiating Amplifier



Colour Code

0	Black	2	Red	4	Yellow	6	Blue	8	Grey
1	Brown	3	Orange	5	Green	7	Violet	9	White

Preferred Values

10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82

11 SOIL MECHANICS

11.1 Soil Classification

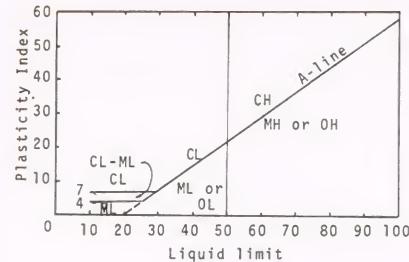
1. Size Classification

CLASSIFICATION	M.I.T. Size limits mm	B.S. Sieves used for separation mm
Gravel	Coarse	20 — 20
	Medium	6 — 6.3
	Fine	2 — 2
Sand	Coarse	0.6 — 0.6
	Medium	0.2 — 0.212
	Fine	— 0.063
Silt	Coarse	— 0.02
	Medium	— 0.006
	Fine	— 0.002
Clay		

2. Casagrande Soil Classification, fine grained (50% or more passing B.S. No.200 sieve)

Silts and clays (Liquid limit less than 50)	Inorganic silts, silty or clayey fine sands, with slight plasticity	ML
	Inorganic clays, silty clays, sandy clays of low plasticity	CL
	Organic silts and organic silty clays of low plasticity	OL
Silts and clays (Liquid limit greater than 50)	Inorganic silts of high plasticity	MH
	Inorganic clays of high plasticity	CH
	Organic clays of high plasticity	OH
Highly organic soils	Peat and other highly organic soils	Pt

M silt L low plasticity
C clay CH high plasticity
O organic OH



3. Volume-weight Relationships

Vol.	Weight
1	Air
e	Water
Se	0
Se	Se γ_w
1	Solids
	$G_s \gamma_w$

$$n = \frac{e}{1+e} \quad e = \frac{n}{1-n}$$

$$w = \frac{Se}{G_s}$$

$$\gamma = \frac{G_s(1+w)}{1+e} \gamma_w$$

$$\gamma' = \frac{G_s - 1}{1+e} \gamma_w = \gamma_{sat} - \gamma_w$$

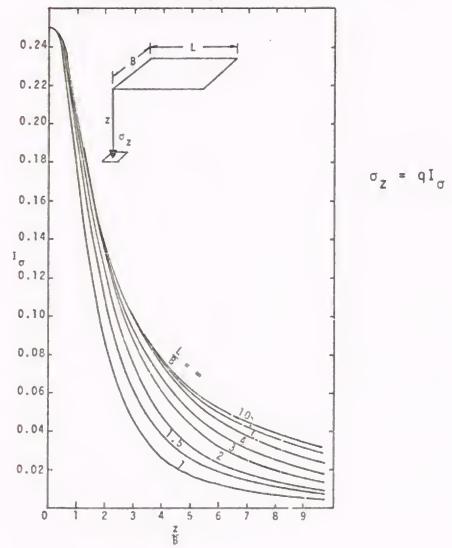
4. Stratigraphic Table

Subdivisions of Quaternary

Era	Period	Epoch	Relative Climate	U.K. Name
Cenozoic	Quaternary	Recent Pleistocene	Warm (current)	Flandrian (Holocene)
	Tertiary	Pliocene Miocene Oligocene Eocene Paleocene	Cold Warm Cold Warm Cold Warm Cold Warm Cold	Devensian Ipswichian Wolstonian Hoxian Anglian Cromerian Beestonian Pastonian Baventian Antian Thurnian Ludhamian Waltonian
Mesozoic	Cretaceous Jurassic Triassic			
Paleozoic	Permian Carboniferous Devonian Silurian Ordovician Cambrian			
Precambrian				

11.2 Stresses and Displacements in Elastic Half-space

1. Vertical stress at depth z below corner of uniformly loaded rectangle



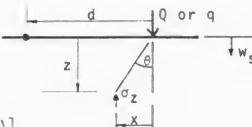
$$\sigma_z = q I_\sigma$$

2. Boussinesq

- (a) Point load Q at surface

$$\sigma_z = \frac{3Q}{2\pi z^2} \cos^5 \theta$$

$$w_s = \frac{Q(1+\nu)}{2\pi z E} \cos \theta [\cos^2 \theta + 2(1-\nu)]$$



- (b) Line load q at surface

$$\sigma_z = \frac{2q}{\pi z} \cos^4 \theta$$

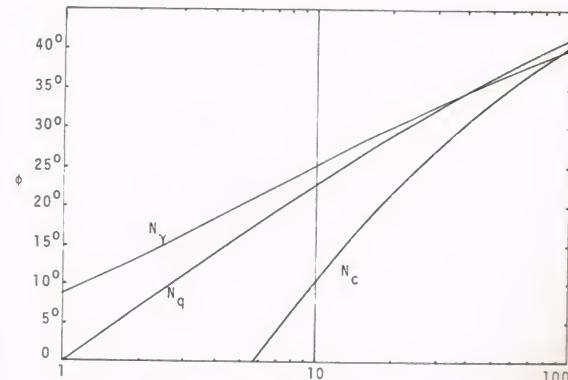
$$w_s = \frac{2q(1-\nu^2)}{\pi E} \ln\left(\frac{d}{z}\right) \text{ where displacement at } d \text{ is assumed } = 0 \quad (d \gg x)$$

3. Surface displacement of uniformly loaded rectangle

L/B Ratio	I_δ		
	Centre	Corner	Average
1	1.12	.56	.95
1.5	1.36	.68	1.15
2	1.53	.76	1.30
3	1.78	.89	1.53
4	1.96	.98	1.70
5	2.10	1.05	1.83
7	2.33	1.16	2.04
10	2.53	1.27	2.25
20	2.95	1.47	2.64
30	3.23	1.61	2.88
50	3.54	1.77	3.22
100	4.01	2.00	3.69
Circle	1.00	Edge .64	.85

$$w_s = qB \frac{1-\nu^2}{E} I_\delta$$

11.3 Terzaghi Bearing Capacity Factors

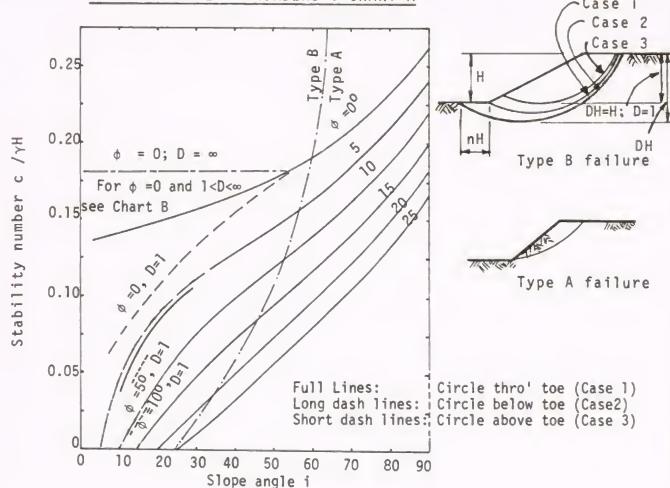


$$\left. \begin{aligned} \text{Strip footing} \quad q &= cN_c + \gamma DN_q + .5Y BN_Y \\ \text{Square footing} \quad q &= 1.3 cN_c + \gamma DN_q + .4Y BN_Y \\ \text{Circular footing} \quad q &= 1.3 cN_c + \gamma DN_q + .3Y BN_Y \end{aligned} \right\} B = \text{FOOTING WIDTH}$$

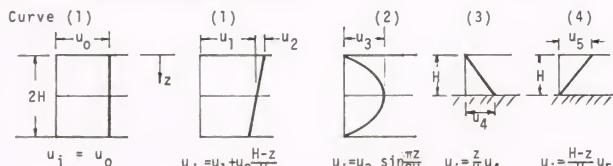
Note: Reduce c and $\tan \phi$ to two thirds of measured values for local shear

11.4 Slope Stability

TAYLOR STABILITY NUMBERS : CHART A



11.5 Consolidation-Time Curves



$$\text{For curve (1): } U < .60 \quad T = \frac{\pi}{4} U^2$$

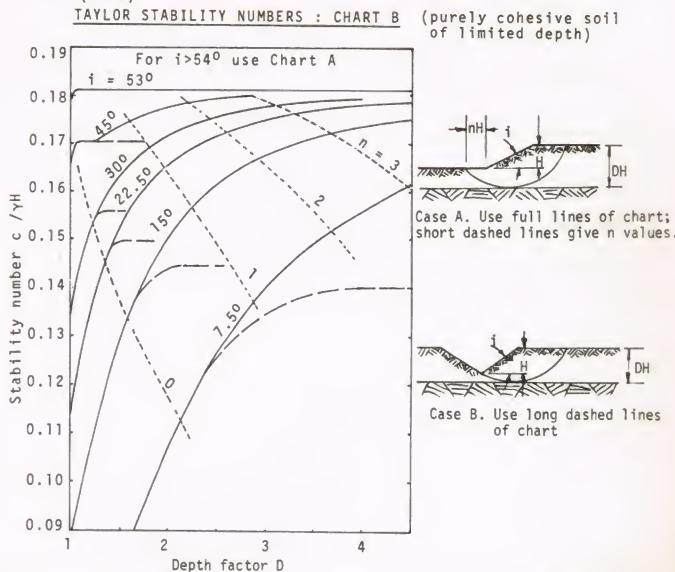
$$U > .60 \quad T = -.933 \log_{10}(1-U) - 0.085$$

$$T_{50} = .197$$

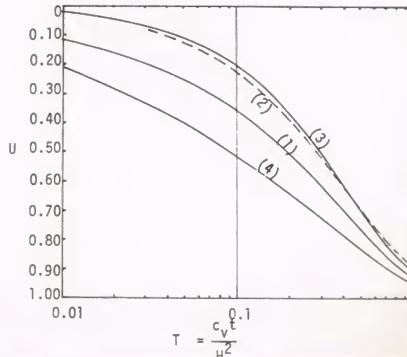
$$T_{90} = .848$$

11.4 (cont)

TAYLOR STABILITY NUMBERS : CHART B

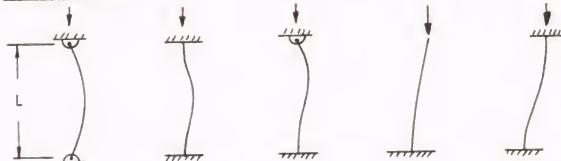


11.5 (cont)



12. STRUCTURES

Buckling Loads



Buckling Load:

$$\frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, \frac{2.045\pi^2 EI}{L^2}, \frac{\pi^2 EI}{4L^2}, \frac{\pi^2 EI}{L^2}$$

Effective Length:

$$L, 0.5L, 0.699L, 2L, L$$

Beams bent about principal axis

w is load/unit length	end slope	maximum deflection Δ
	ML/EI	$ML^2/2EI$
	$WL^2/2EI$	$WL^3/3EI$
	$\omega L^3/6EI$	$\omega L^4/8EI$
	$ML/2EI$	$ML^2/8EI$
	$WL^2/16EI$	$WL^3/48EI$
	$\omega L^3/24EI$	$5\omega L^4/384EI$
	$\theta_b = \frac{Wac^2}{2LEI}$ $\theta_a = \frac{L+b}{L+a} \cdot \theta_b$	$\frac{Wac^3}{3LEI}$

Fixed End Moments

LH end conditions moment-shear	RH end conditions shear-moment	maximum deflection Δ	maximum deflection position c
$\frac{wL^2}{T2}$	$\frac{wL}{2}$	$\frac{wL^2}{2} \uparrow \left(\frac{wL^2}{T2}\right)$	$\frac{wL^4}{384EI}$ $\frac{L}{2}$
$\frac{WL}{8}$	$\frac{W}{2}$	$\frac{W}{2} \uparrow \left(\frac{WL}{8}\right)$	$\frac{WL^3}{T92EI}$ $\frac{L}{2}$
$\frac{Wa^2}{L^2}$	$\frac{Wa^2(L+2a)}{L^3}$	$\frac{(Wa^2b)}{L^2} \uparrow \left(\frac{Wa^2b}{L^2}\right)$	$\frac{2Wa^2b^3}{3EI(L+2b)^2}$ $a < b$
$\frac{6EI}{L^2}$	$\frac{12EI}{L^3}$	$\frac{12EI}{L^3} \downarrow \left(\frac{6EI}{L^2}\right)$	1 -
$\frac{Mb}{L^2(2a-b)}$	$\frac{M}{L} \downarrow \frac{Ma}{L} \uparrow \left(\frac{Ma}{L}(2b-a)\right)$	-	-
$\frac{wL^2}{30}$	$\frac{3wL}{20}$	$\frac{7wL}{20} \uparrow \left(\frac{wL^2}{20}\right)$	$\frac{wL^4}{764EI}$ 0.475L
$\frac{3WL}{16}$	$\frac{11W}{16}$	$\frac{5W}{16} \uparrow 0$	$\frac{2WL^3}{215EI}$ 0.447L
$\frac{Wab(L+b)}{2L^2}$	$\frac{Wb}{L+C} + \frac{M}{L-C}$	$\frac{Wa-M}{L-L} \uparrow 0$	$\frac{Wa^2b}{6EI\sqrt{2L+b}}$ $b > 0.4142L$
$\frac{wL^2}{8}$	$\frac{5wL}{8}$	$\frac{3wL}{8} \uparrow 0$	$\frac{wL^4}{185EI}$ 0.422L

Relations with elastic constants

$$G = E/(2(1 + \nu)) \quad K = E/(3(1 - 2\nu))$$

$$\text{Simple bending } \frac{M}{t} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\text{Torsion of circular section } \frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Beam stiffness Coefficients

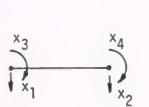
In the following the F's can be axial or shear forces, or, bending or torsional couples corresponding to the mode of deformation.

All beam and frame stiffness matrices may be built up from the following components of each beam element.

(a) axial stiffness  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

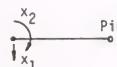
(b) torsional stiffness  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{GJ}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(c) bending stiffness and lateral deflection stiffness in one plane



giving $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & -12 & 6l & 6l \\ -12 & 12 & -6l & -6l \\ 6l & -6l & 4l^2 & 2l^2 \\ 6l & -6l & 2l^2 & 4l^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

In each case if one end is fixed and considered as a reaction, its deflections and forces may be ignored with a corresponding reduction of the stiffness matrix. Another possible form of reaction for case (c) occurs if the reaction end is pinned. Then the stiffness matrix components for the other end are given by

(c)(i)  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 3 & 3l \\ 3l & 3l^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

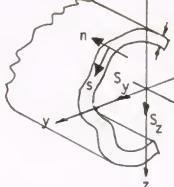
or

(c)(ii)  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EI}{l^2} \begin{bmatrix} 3 & -3l \\ -3l & 3l^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

A general plane frame element will have components (a) and (c) and require a 6×6 stiffness matrix. A general space frame element will require components of (a), (b) and (c) - the latter twice for two planes of bending - and will, require a 12×12 stiffness matrix. The three modes of deflection (a), (b), (c) are orthogonal and may be combined into larger matrices with 0's in all unspecified positions. Space frame elements will, in general, have different values of I in the two principal planes of bending.

Shear

Shear flow per unit length of wall resulting from the applied shear forces S_z, S_y is



$$q = t\tau_{xs} = \frac{(-S_z)}{I_{yy}I_{zz} - (I_{yz})^2} \left[I_{yy} \int_A zdA - I_{yz} \int_A ydA \right] + \frac{(-S_y)}{I_{yy}I_{zz} - (I_{yz})^2} \left[I_{zz} \int_A ydA - I_{yz} \int_A zdA \right]$$

The resultant force from this shear flow acts through the SHEAR CENTRE.

Torsion

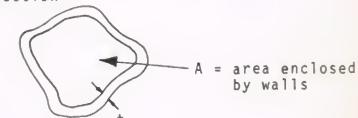
$$\text{For a circular section } \frac{T_x}{J} = \frac{\tau_{\theta x}}{r} = \frac{G\theta}{L}$$

$$J = \frac{\pi D^4}{32} \text{ for a solid section}$$

$$= \frac{\pi}{32} [D_{\text{outer}}^4 - D_{\text{inner}}^4] \text{ for a hollow section}$$

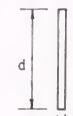
For a thin walled closed section

$$\begin{aligned} T_x &= 2Aq \\ &= \frac{4A^2G}{\int_0^t s ds} \cdot \frac{\theta}{L} \end{aligned}$$



For a thin rectangular section

$$\begin{aligned} T_x &= \frac{dt^2}{3} \tau_{zx} \max \\ &= \frac{dt^3 G}{3} \cdot \frac{\theta}{L} \end{aligned}$$



Asymmetric Bending

in terms of general axes

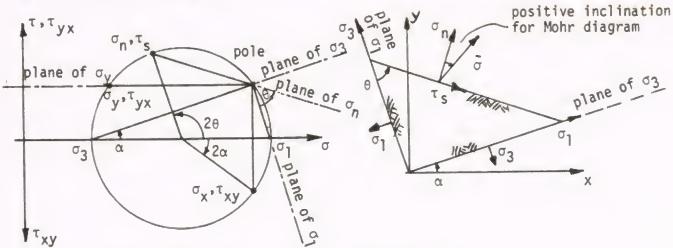
$$\sigma_{xx} = \frac{P_x}{A} + \frac{M_y(zI_{zz} - yI_{yz})}{I_{yy}I_{zz} - (I_{yz})^2} - \frac{M_z(yI_{yy} - zI_{yz})}{I_{yy}I_{zz} - (I_{yz})^2}$$

When the principal axes m,n lie in directions y,z, then:

$$\sigma_{xx} = \frac{P}{A} + \frac{nM_m}{I_{mm}} - \frac{mM_n}{I_{nn}}$$

Stress and strain transformations

Mohr circle of stresses



Equilibrium of prism in σ_1, σ_3 directions gives

$$\sigma_1 \cos \theta = \sigma_n \cos \theta + \tau_s \sin \theta$$

$$\sigma_3 \sin \theta = \sigma_n \sin \theta - \tau_s \cos \theta$$

$$\text{Then: } \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta = \sigma_n = \frac{\sigma_1}{2}(1+\cos 2\theta) + \frac{\sigma_3}{2}(1-\cos 2\theta)$$

$$= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$(\sigma_1 - \sigma_3) \sin \theta \cos \theta = \tau_s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

$$\text{Also: } \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \tau_{\max}; \quad \sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \tau_{\max}; \quad \tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\text{and: } \tan 2\alpha = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y}$$

Two-dimensional strain system

$\epsilon_x, \epsilon_y, \epsilon$ are direct strains 'corresponding' to $\sigma_x, \sigma_y, \sigma$

$\frac{\gamma_{xy}}{2}, \frac{\gamma}{2}$ are shear strains 'corresponding' to τ_{xy}, τ

Three-dimensional stress system

If the principal stresses are $\sigma_1, \sigma_2, \sigma_3$, the principal shear stresses are $(\sigma_1 - \sigma_2)/2, (\sigma_2 - \sigma_3)/2$ and $(\sigma_3 - \sigma_1)/2$.

Strain energy per unit volume U may be expressed as

$$U = (\sigma_1 + \sigma_2 + \sigma_3)^2 / 18K$$

$$+ [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 12G$$

13. SYMBOLS INDEX

GREEK ALPHABET

A, α alpha	H, η eta	N, ν nu	T, τ tau
B, β beta	θ, Θ theta	Ξ, ξ xi	Υ, υ upsilon
Γ, γ gamma	I, ι iota	O, \circ omicron	ϕ, Φ phi
Δ, δ delta	K, κ kappa	Π, π pi	X, χ chi
E, ϵ epsilon	Λ, λ lambda	P, ρ rho	Ψ, ψ psi
Z, ζ zeta	M, μ mu	Σ, σ sigma	Ω, ω omega

MATHEMATICAL SYMBOLS

L[]	- Laplace Transform
\triangle	- defined as
Σ	- repeated summation
Π	- repeated multiplication
∂	- partial differential
a	- modulus
∇	- Laplace differential, Del, Nabla
\hat{a}	- vector
$\hat{\hat{a}}$	- unit vector
\perp	- 'at right angles to'
.	- scalar (dot) product
x	- vector (cross) product
Re	- real part of complex number
Im	- imaginary part of complex number

Symbol	Page No. of use		Recommended S.I. Unit
a	51	velocity of sound	m/s
a	38	lattice parameter	m
a	41	crack length	m
a	60	area	m ²
a	37	acceleration	m/s ²
A	48, 50	area	m ²
A	38	Atomic weight	-
A	47	availability function (non-flow)	kJ
B	47	availability function (flow)	kJ
B	60	magnetic flux density	T
B	66, 67	breadth of footing	m

* of specified fluid			
c	67	cohesion	kN/m ²
c	49	velocity	m/s
c _p	47	specific heat	kJ/(kgK)
c _v	69	coefficient of consolidation	m ² /s
c _C	60	capacitance	F
C	39	concentration	mol/m ³
C	50	Chézy coefficient	(m ²)/s
c _d	50, 51	discharge coefficient	-
c _d	51	drag coefficient	-
d	38	interatomic spacing	m
d ₁	38	interplanar spacing	m
d	50	depth of flow	m
d ₁ , d ₂	50	" " before jump, after jump	m
D	51, 54	diameter	m
D	39	diffusion coefficient	m ² /s
D	67	depth of overburden	m
D	68, 69	depth factor	-
e	65	void ratio	-
E	41, 70	Young's Modulus	N/m ²
ΔE	38	energy difference	J
f	61	supply frequency	Hz
f	54	friction factor	-
f	51	wall shear stress coefficient	-
f	47	specific Helmholtz free energy function	kJ/kg
F	47	Helmholtz free energy function	kJ
F, F ₁	50, 51	Froude Number, before jump	-
F	60	force	N
g	47	specific Gibbs free energy function	kJ/kg
G _s	65	specific gravity of solids	-
G _s	4/	Gibbs free energy function	kJ
G	41, 72	modulus of rigidity	N/m ²
G ₁ , G ₂ , G	55	transfer function	-
h _f	51, 54	frictional head loss	m*
h	47	specific enthalpy	kJ/kg
h	51	heat transfer coefficient	W/(m ² K)
H	47	enthalpy	kJ
H	60	magnetising force magnetic field strength	A/m
H	55	transfer function	-
I	60	current	A
I	35, 70	second moment of area	m ⁴
I	35	moment of inertia	kgm ²
J	39	diffusion flux	kg/(m ² s)
k	48, 51	thermal conductivity	W/(mK)
k	54	surface roughness	µm
k	35, 49	radius of gyration	m
k _d	61	distribution factor, pitch factor of winding	-
k _d	55	gain constant	-
K	41	stress intensity factor	MNm
K	41, 72	bulk modulus	N/m ²
ΔK	41	stress intensity range	MNm ^{1/2}
K _p	47	equilibrium constant	(atmos) ^{1/n}
&	51	length	m
&	60	length of conductor	m
L	51	characteristic length	m
L	60	inductance	H
m	47	mass	kg
m	51, 54	mean hydraulic radius	m
m̄	47, 50	mass flowrate	kg/s

M	70	moment	Nm
M	51	Mach number	-
n	65	porosity	-
n	61	speed of rotation	r/min
n	50	Manning roughness coefficient	-
n	38	atoms per unit cell	-
ni	41	total cycles at stress amplitude	-
NP	41	total cycles to failure at strain amplitude	-
N _i	41	" " " " stress	-
N _i	60, 61	number of turns	-
N	7, 38	Avogadro's number	kg/(kgmol)
N	47	cycles per unit time	Hz
N ^c	39	total number of atoms	-
p	30	probability	-
p	47	mean effective pressure	N/m ²
P _B	61	number of pole pairs	-
P _P	47	thermodynamic probability or No. of quantum states	-
P _i	47	engine indicated power	W
q _i	66, 67	surface normal stress	kN/m ²
q	48	heat flowrate per unit area emissive power	W/m ²
q _b	48	emissive power for black body	-
Q _b	39	activation energy	kJ
Q	47	heat (input + ve)	m ³ /s
Q	49	volumetric flowrate	-
Q	41	crack shape factor	-
Q	60	charge	C
r	41	r.m.s. of stress	MN/m ²
R	60, 61	resistance, resistance per phase	Ω
R	50	hydraulic radius	m
R	47	characteristic gas constant	kJ/(kgK)
R ₀	47	Universal gas constant	kJ/(kgmolK)
s	47	specific entropy	kJ/(kgK)
s	61	fractional slip	-
s	65	degree of saturation	-
S	47	entropy	kJ/K
S	50	channel slope in uniform flow	-
S _f	50	friction slope	-
S _f	50	invert slope	-
t ^o	37	time	s
T	68	time factor	-
T	61	torque	Nm
T	50	water surface width	m
T	47	temperature (absolute)	K
ΔT	48	temperature difference	°C
T	43	glass transition temperature	K
u ₉	47	specific internal energy	kJ/kg
u	37	velocity	m/s
u	68	pore pressure	kN/m ²
U	68	degree of consolidation	-
U	47	internal energy	J
v	47	specific volume	m ³ /kg
v	37	velocity	m/s
v ₀	47	molar volume	m ³ /(kgmol)
V	60	voltage	V
V	38, 49	volume	m ³
V	38	volume of unit cell	-
V	50, 51	velocity	m/s
V _s	47	cylinder swept volume	m ³

w	65	water content	-
w _s	66,67	surface displacement	m
w	47	work (output,+ve)	kJ
X	61	leakage reactance per phase	Ω
y ₀	49	change in specific energy through a machine	J/kg
z	49	potential head	m*
Z	61	number of armature conductors	-
Z	62	impedance	Ω
α	41	coefficient of linear expansion	$\mu\text{m}/(\text{mK})$
α	60	resistance coefficient	Ω/K
B	51	coefficient of volumetric expansion	K^{-1}
Y	65	unit weight of soil	kN/m ³
Y	47,50	specific heat ratio	kN/m ³
Y _w	65	unit weight of water	kN/m ³
Y _{sat}	65	unit weight of saturated soil	kN/m ³
Y	65	submerged unit weight of soil	-
ϵ	54	relative roughness	F/m,-
$\epsilon_0 \cdot \epsilon_r$	60	permittivity, free space, relative	-
ϵ	48	emissivity	-
ϵ_p	41	plastic straining range	$\mu\text{m}/\text{m}^2$
μ	49	viscosity (dynamic)	mNs/m^2
$\mu_0 \cdot \mu_r$	60	permeability of free space, relative	H/m, -
μ	49,51	dynamic viscosity	mNs/m^2
μ	37	coefficient of friction	-
v	72,41,66,67	Poisson's ratio	-
ξ	58,23	damping ratio	-
ρ_0	60	resistivity	Ωm
ρ	41,42,43,5	density	$\text{kg}/(\text{m}^3 \text{K}^4)$
σ	48	Stefan-Boltzmann constant	$\text{W}/(\text{m}^2 \text{K}^4)$
σ	51	surface tension in contact with air	N/m
σ_y	41	proof or yield stress	N/m ²
σ_f	41	ultimate (failure) stress	N/m ²
τ	49,51	shear stress	N/m ²
ϕ	67	friction angle of soil	o
Φ	60,61	magnetic flux, flux per pole	Wb
w	70,71	load per unit length	N/m
w	51	angular velocity	rad/s
w _n	58	natural frequency	rad/s
w _o	23,62	natural frequency	rad/s
w _d	59	damped natural frequency	rad/s

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